

# THE SUPER-EFFICIENCY MODEL AND ITS USE FOR RANKING AND IDENTIFICATION OF OUTLIERS

Kristína Kočišová<sup>1</sup>, Iveta Palečková<sup>2</sup>

<sup>1</sup>Technical University of Košice, Faculty of Economics, Nemcovej 32, 040 01 Košice, Slovak Republic

<sup>2</sup>Silesian University, School of Business Administration, Univerzitní náměstí 1934/3, 733 40 Karviná, Czech Republic

## Abstract

KOČIŠOVÁ KRISTÍNA, PALEČKOVÁ IVETA. 2017. The Super-efficiency Model and its Use for Ranking and Identification of Outliers. *Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis*, 65(4): 1371–1382.

This paper employs non-radial and non-oriented super-efficiency SBM model under the assumption of a variable return to scale to analyse performance of twenty-two Czech and Slovak domestic commercial banks in 2015. The banks were ranked according to asset-oriented and profit-oriented intermediation approach. We pooled the cross-country data and used them to define a common best-practice efficiency frontier. This allowed us to focus on determining relative differences in efficiency across banks. The average efficiency was evaluated separately on the “national” and “international” level. Based on the results of analysis can be seen that in Slovak banking sector the level of super-efficiency was lower compared to Czech banks. Also, the number of super-efficient banks was lower in a case of Slovakia under both approaches. The boxplot analysis was used to determine the outliers in the dataset. The results suggest that the exclusion of outliers led to the better statistical characteristic of estimated efficiency.

Keywords: super-efficiency, asset-oriented intermediation approach, profit-oriented intermediation approach, outliers, banking, Czech Republic and Slovakia

## INTRODUCTION

Financial system consists of a financial market, financial institutions, financial instruments, creditors, debtors and financial transactions. The well-functioning financial system is crucial to economic health. Financial markets perform the essential economic function of channelling funds from households, firms, and government that have saved surplus funds by spending less than their income to those that have a shortage of funds because they wish to spend more than their income. The funds flow from lender-savers to borrower-spenders could be done via two main channels: direct or indirect finance. Under direct finance (sometimes called Market-oriented financial system, Capital-market-oriented financial system or M-system), borrowers borrow funds directly from lenders in financial markets by selling them financial instruments, which are claims on the borrower's

future income or assets. Under indirect finance (sometimes called Bank-oriented financial system or B-system) the funds are moved from lenders to borrowers by the financial intermediaries, that stands between the lender and the borrower and helps transfer funds from one to the other. The financial intermediaries do this by borrowing funds from the lenders and then using these funds to make loans to borrowers. (Mishkin, 2009)

The world's financial system plays an important role in capital allocation. Its size can be measured by various methods. Basic characteristics of the financial systems of individual countries may be based on selected indicators of financial markets (e.g. the volume of debt securities or equity market capitalization) and financial intermediaries (e.g. the volume of total assets, deposits or loans). Selected indicators are usually compared in relation to Gross Domestic Product (GDP) or the analogous indicator

so that it is possible to compare the financial systems with different size. The importance of allocating funds via various channels (direct or indirect finance) in individual financial systems differs substantially. The bank-oriented financial market is more common. Its essence is that a critical part of financial transactions is passing through the commercial banks. Therefore it is very important to study their efficiency and try to find out how the commercial banks could reduce their inefficiency in the process of transformation of funds. Inefficient banking sector together with poor financial infrastructure, which is not enhanced by strong legislation, could slow economic growth of the country. Nowadays, when many banks operate on the international basis, inefficiency in the banking sector in one country can be transmitted as financial contagion to other countries.

Most existing studies about the efficiency in the Slovak and Czech banking sector employ Data Envelopment Analysis (DEA), Stochastic Frontier Approach (SFA), or traditional financial ratios, to analyse it. In the case of DEA, the authors prefer to use basic models under the assumption of constant or variable return to scale to measure technical efficiency. Some authors also use information about input or output prices and try to analyse cost, revenue or profit efficiency. The disadvantage of these models is that the efficient units in a sample share the same efficiency score equal to one. This problem can be removed by solving so-called super-efficiency models, where the researcher is able to distinguish among the efficient units in the sample. The advantage of super-efficiency models compared to basic models is, that they allow to analyse not only these units which were inefficient, but also analyse these units which were marked as efficient under the basic DEA models. Another advantage of super-efficiency models is that we can identify the outliers which can deform the shape of efficiency frontier and therefore it is better to exclude them from the analysis. After re-estimation of super-efficiency, we can obtain a dataset with better statistical characteristic (e.g. lower variance, no extreme values and so on). The super-efficiency calculated this way also helps in calculating correlation coefficients and in using of DEA results in regression analysis, as the efficient units don't have the same score equal to one and don't contain the outliers.

In the conditions of Slovak and Czech banking sector, there is only a small number of studies which used the super-efficiency models to calculate efficiency. Therefore the aim of the paper is to describe the methodology from the theoretical point of view, apply the super-efficiency method based on SBM (Slacks-Based Measure) model to measure efficiency, to rank efficient units and to identify outliers. The study is organised as follow: Section 2 shows literature review of existing studies about the efficiency of the banking industry in our conditions. Section 3 introduces

the non-radial non-oriented super-efficiency SBM model under the assumption of variable return to scale. In the next section, Section 4, we describe the data and our empirical results. The last section brings the main findings in form of conclusion.

### Literature review

There are many existing studies that use basic DEA models to measure technical efficiency, or some extension of basic models to measure cost, profit or revenue efficiency in the Czech and Slovak banking industry. Boďa and Zimková (2015) measured technical efficiency of eleven commercial banks in three sub-periods: 2000–2003, 2004–2008 and 2009–2011. The technical efficiency was also analysed in the work of Palečková (2015), who found the increase in the average efficiency of the Slovak commercial banks during the period 2004–2013. Iršová and Havránek (2011) used information about input and output prices and found a low average cost and profit efficiency in Slovakia during the years 1995–2006. Stavárek and Řepková (2012) found that average technical efficiency increased in the Czech banking sector within the period 2001–2010. Polouček *et al.* (2004) estimated technical efficiency and profitability in selected banking sectors in CEC (Poland, the Czech Republic, Hungary, and Slovakia) and two European Union countries (Finland and Belgium) in 2000 and 2001. Authors found that the Czech banking sector was marked as the most efficient. Svitálková (2014), who measured and compared the technical efficiency of bank system in selected countries in the European Union (Czech Republic, Slovakia, Austria, Poland, Hungary, Slovenia) during the period 2004–2011, concluded that Czech banking sector was between the best efficient countries and the banking sector in Slovakia had the worst performance within the analysed countries. These findings were confirmed also by Kočíšová (2014) who found that the Czech banks were more cost, revenue and profit efficient than Slovak ones during the period 2009–2013. The cost and revenue efficiency were analysed by Pančurová and Lyócsa (2013), who estimated efficiencies and their determinants for a sample of 11 Central and Eastern European Countries over the 2005–2008 period. They found out no dramatic changes in the average efficiencies during the analysed period, although cost efficiency declined slightly and revenue efficiency increased. The average cost efficiency was higher for the Baltic countries and the Czech Republic. Lower values were observed for Romania and Hungary.

One of the few studies which applied the super-efficiency models in our condition is study prepared by Zimková (2014). She estimated the technical efficiency and the super-efficiency on the sample of 16 banking institutions in Slovakia in 2012. She found out that the level of efficiency differs from one bank to another. More than half of institutions were found technically efficient by applying basic input-oriented DEA

model under the assumption of a variable return to scale. Consequently, the input-oriented super-efficiency model SBM model provided the list of the super-efficient banks in Slovakia. Zimková (2015) also used this model to evaluate super-efficiency of the insurance companies in Slovakia.

One of the important aspects in the process of efficiency measurement is to define input and output variables. The choice of inputs and outputs is usually a critical part of analysis. Several approaches were developed in the empirical literature that define the relationship between inputs and outputs in the behaviour of financial institutions. Firstly, an intermediation approach was introduced by Sealey and Lindley (1977) where banks are characterised as financial intermediaries. Thus, the intermediation approach assumes that the main aim of a commercial bank is to create output, defined as loans and investment or other assets, whilst using liabilities (including deposits), labour and capital as inputs (Boďa and Zimková, 2015). There are two orientations in the application of the intermediation approach regarding the measure of the intermediation factors: the asset-oriented and profit-oriented intermediation approach. Berger and Humphrey (1997) showed that under the asset-oriented intermediation approach, banks are considered only as financial intermediaries between liability holders and those who receive bank funds. Loans and other assets are considered to be bank outputs; deposits and other liabilities are inputs to the intermediation process. Profit-oriented intermediation approach was defined by Berger and Mester (2003), who reported that it can help capture the objective of maximising profits by including costs and revenues. Boďa and Zimková (2015) described that the profit-oriented intermediation approach attempts to capture final monetary effects of financial intermediation, in which interest expenses and/or non-interest expenses are found as inputs and interest income and/or non-interest income are used as outputs. Such a specification retains the minimising feature of inputs and the maximising feature of outputs.

The second approach is the production approach which was pioneered in the study of Benston (1965) and it was also presented in the study of Sherman and Gold (1985). Under this approach, banks are characterised as service producers aiming at minimising operating costs (Ahn and Le, 2014). Inputs under this approach include only physical variable such as labour, premises and fixed assets, space or information system and their associated costs. Interest expenses are excluded from this approach since the main focus is on operating processes. The output factors could include deposits, loans and securities. Boďa and Zimková (2015) stated that this variant of the production approach that considers deposits to be an output together with loans and the interest income is called the service-oriented approach.

The value-added approach considers that all liability and asset categories have some output characteristics. The categories having substantial value added are employed as the important outputs. Others are treated as representing mainly unimportant outputs, intermediate products, or inputs, depending on the specifics of the category. The value-added approach explicitly uses operating cost data (Berger and Humphrey, 1997). As Boďa and Zimková (2015) described the application of this approach requires a more sensitive analysis of individual processes that are carried out by the commercial banks under evaluation.

Beside the main mentioned approaches to defining input and output variables, there exist many variations of them. The selection of variables in each study depends on the research question, data and analysed period. The selection of variables in a case of bank branches can be different as variables used in the evaluation of the efficiency of banks, or banking sectors. As well as a selection of variables nowadays, when the banks offer a range of services based on the usage of informational technologies may be different than in the last century.

## MATERIALS AND METHODS

DEA is a method for measurement of the relative efficiency of Decision-Making Units (DMU), using the same multiple inputs to produce multiple outputs. In recent years in our conditions (the Czech Republic and Slovakia), this method became increasingly popular to measure effectiveness in the service sector, e.g. financial services, health services (e.g. Sendek *et al.* 2015, Stefko *et al.* 2016), education, transport, hotel services, and so on.

DEA is used to establish a best practice group of units and to determine which units are inefficient compared to best practice group as well as to show the magnitude of the inefficiencies present. The basic DEA models, input or output oriented, allow calculating with an assumption of constant or variable return to the scale. The input-oriented models bring a recommendation for inefficient units to achieve efficiency in form of reduction on the input side. Output oriented models required to achieve efficiency increase on the output side. The model with the constant return to the scale assumption is called CCR (Charnes, Cooper and Rhodes) model. The assumption of a constant return to scale can be accepted only if the units operate under the condition of their optimal size. Imperfect competition, financial constraints, control steps and other factors are conducive to the fact that units don't operate under their optimal size. Therefore, to overcome this problem has been developed DEA model, which allows calculating with variable returns to scale. This model is called a BCC model (Banker, Charnes, Cooper). The "relative" efficiency calculated by the basic DEA models can achieve values from 0 to 1, thus enabling easy comparison, where 1 represents an efficient unit relative to others

in the sample, and a unit with a score less than 1 is defined as inefficient. These DEA models are based on the Pareto optimality for efficient production. Pareto optimality states that a unit isn't efficient if it is possible to raise an output without raising any of the inputs and without lowering any other output; similarly, a unit isn't efficient if it is possible to lower an input without decreasing any of the outputs and without increasing any other input. (Cooper, Seiford, and Tone, 2007)

Basic DEA models suffer from tied ranks because the efficient units in a sample share the same score of one. This problem can be removed by solving so-called super-efficiency. Through this model the researcher is able to distinguish among the efficient units in the sample and rank them. In this study the units of analysis are banks. Consider  $n$  banks ( $DMU_j, j = 1, 2, \dots, n$ ), each consumes  $m$  different inputs ( $x_{ij}, i = 1, 2, \dots, m$ ) to produce  $s$  different outputs ( $y_{rj}, r = 1, 2, \dots, s$ ). We used the assumption of a variable return to scale (as commercial banks in the real world usually don't operate under their optimal size) combined with a non-radial and non-oriented super-efficiency SBM model. The non-oriented aspect of the model captures the desire to improve both the inputs and outputs simultaneously. The non-radial aspect captures that the movements on efficiency frontier will be not only radial (proportional). For the movement on the efficiency frontier there is necessary also non-radial movement which is expressed via the values of slacks (non-radial input excess or non-radial output shortfalls).

The discussion about the super-efficiency is taken under the assumption that the production unit  $DMU(x_o, y_o)$  is SBM-efficient, i.e. it is strongly efficient under the SBM model. The detail definition of SBM model can be find in Cooper, Seiford, and Tone (2007). Compared to basic DEA models (CCR and BCC), the SBM model in evaluating efficiency captures the non-radial slacks directly. The purpose of SBM model is to minimise the input and output slacks by solving linear program problem.

Based on the preconditions described in detail in Cooper, Seiford, and Tone (2007) the super-efficiency of  $DMU(x_o, y_o)$  under the assumption of variable return to scale was defined as the optimal objective function  $\delta^*$  from the following program:

$$\delta^* = \min_{\bar{x}, \bar{y}, \lambda} \frac{\frac{1}{m} \sum_{i=1}^m \bar{x}_i / x_{io}}{\frac{1}{s} \sum_{r=1}^s \bar{y}_r / y_{ro}} \quad (1)$$

Subject to

$$\bar{x} \geq \sum_{j=1, j \neq o}^n \lambda_j x_j$$

$$\bar{y} \leq \sum_{j=1, j \neq o}^n \lambda_j y_j$$

$$\sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0, \forall j$$

$$\bar{x} \geq x_o, \text{ and } \bar{y} \leq y_o$$

$$\bar{y} \geq 0, \lambda \geq 0$$

The super-efficiency score  $\delta^*$  is not restricted to the interval  $[0, 1]$ . Its value is always non-negative and for super-SBM technically efficient units is not smaller than 1.

There are two interpretations of this super-efficiency score. According to Zimková (2015) the higher value means the higher technical efficiency of the unit. It can alternatively be used for finding outliers in the dataset. Large values of super-efficiency that seem out of the other series values indicate that the analysed unit should be treated without a doubt as an outlier. Banker and Chang (2006) defined outliers as a few extreme observations often caused by errors in measuring either the inputs or outputs. Since extreme observations determine the production frontier in DEA models, the estimation of the frontier may be sensitive to measurement errors in the sample. If an observation has been contaminated with noise that increases the observed outputs or decreases the observed inputs such that it gets rated as efficient, then it may also enter the reference of other observations and distort their estimated efficiencies. Such outliers may be influential in the estimation results obtained using a conventional DEA models. It is desirable, therefore, to consider a procedure that allows us to identify and remove such outliers.

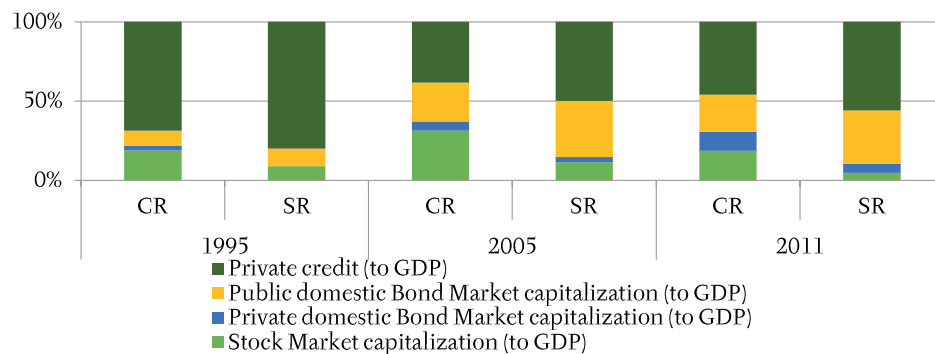
The easiest way how to define outliers is through the boxplot analysis. Another, more sophisticated methods are Timmer's procedure or BG methodology. Timmer (1971) suggested discarding a certain percentage of technically efficient observations from the sample and re-estimating the production frontier using the remaining units. Banker and Gifford (1988) suggested in BG methodology to use the super-efficiency score to identify outliers. Those observations with super-efficiency scores higher than a pre-selected screen should be eliminated. If an efficient observation in an outlier that has been contaminated with noise then it is more likely to have an output (or input) level much greater (smaller) than that of other observations with similar input (or output) levels. Therefore, such outliers are more likely to have a super-efficiency score much greater than one. (Banker and Chang, 2006)

## RESULTS

### The size and structure of Slovak and Czech banking sector

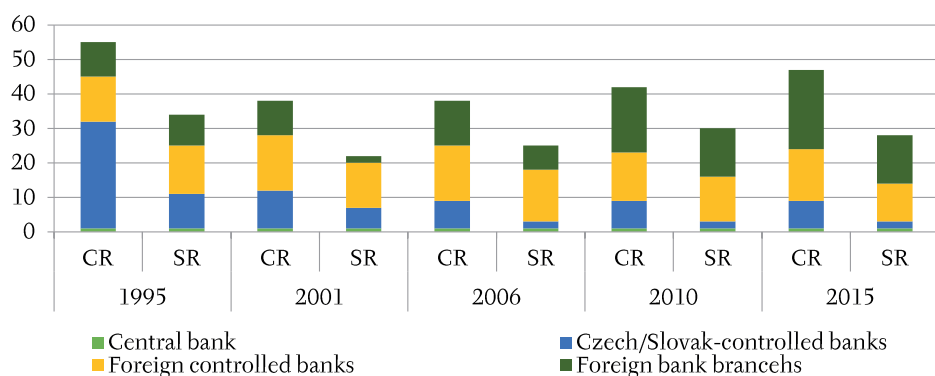
The Slovak and Czech banking sectors are representatives of the bank-oriented financial system, where the financial intermediaries play a crucial role. Following figure (Fig. 1) displays the relative importance of financial intermediaries in our condition based on the selected indicators of financial market and financial intermediaries.





1: Structure of financial system in Slovakia (SR) and Czech Republic (CR)

Source: Source: Authors' calculations based on CESifo (2014)



2: Banking sector in Slovakia (SR) and Czech Republic (CR); number of institutions

Source: Authors' calculations based on [www.nbs.sk](http://www.nbs.sk) and [www.cnb.cz](http://www.cnb.cz)

The structure in figure confirms, that in our conditions are, and used to be bank-oriented financial systems. We can see that the relative importance of other types of financing also increased, but financing via financial intermediaries represented nearly 50% of all resources allocated in the countries in 2011.

Mishkin (2009) indicates that the main financial intermediaries are commercial banks. In our conditions, the term “bank” encompasses several types of banks aside from a central bank. According to the Act on Banks in the Czech Republic and in the Slovakia, a domestic bank can be defined as a joint-stock company based in the country (CR, or SR), accepting deposits from the public and granting loans; licensed by the central bank of the country (the Czech National Bank, or the National Bank of Slovakia). The second type of banks are foreign banks. Foreign banks can operate as a branch upon authorization (license) given by the central bank. Banks coming from the European Union (EU) can operate as a branch without receiving a license from the National Bank of Slovakia or Czech National Bank. Since May 1, 2004, when Slovakia and the Czech Republic joined the EU, the simplified procedure (“the single banking license”) enabled foreign banks licensed within the EU Member States to exercise the freedom to provide services within

the territory of the Slovak or the Czech Republic on a cross-border basis.

Slovak and Czech banking sectors are an example of two stage banking system, which is created by the central bank and the network of commercial banks. Fig. 2 shows the development of the number of banks and their structure in Slovakia and the Czech Republic between the 1995 and 2015. Although the total number of banks in the period did not change significantly, there was a change in the legal form and the ownership structure. Based on the data in figure (Fig. 2), it is clear that after the establishment of the independent Slovak Republic and the Czech Republic in 1993, banks were gradually transferred from the state-owned form to form a joint-stock companies and were gradually privatised so they got into the hands of foreign investors. While in 1995 there were in the Czech banking sector for more than 30 banks without foreign capital participation (in Slovakia there were 10 banks without foreign capital), in 2006, the number dropped to 6 (in the case of Slovakia to 2). With decreasing number of banks without foreign participation, on the other hand, can be seen the growing number of banks with foreign capital, as well as the number of branches of foreign banks.

### Estimation of Slovak and Czech banks efficiency

As can be seen, the commercial banks play a crucial role in the banking sector of Slovakia and the Czech Republic. Therefore it is very important to study their efficiency as it was mentioned in introduction. In our research, we have focused on the evaluation of domestic commercial banks, the foreign controlled branches operated in the area of the Czech Republic and Slovakia were not evaluated. The analysis is based on the data of domestic banks, which comprises more than 75% of total banking assets in both countries. We evaluated efficiency only of universal commercial banks; the specialised banks (e.g. central banks, mortgage banks or savings banks) were not involved in the dataset. The dataset consists of 22 banks (13 from the Czech Republic and 9 from Slovakia) in 2015.

The term "relative" efficiency refers to achieved efficiency of evaluated bank within the group of evaluated banks and under the used criteria. For evaluation of relative efficiency intermediation approach were used. The calculation was done using DEA Solver-Pro software<sup>1</sup>. We decided to use both forms of intermediation approaches in our analysis. We would like to use asset-oriented intermediation approach, as the dominated research in banking area for measurement the economic viability of the banks, and profit-oriented intermediation approach, for evaluating the efficiency of costs and revenues management. After the survey of a number of similar studies, the following set of inputs and outputs for both approaches was applied.

In case of asset-oriented intermediation approach (AOIA) we used two inputs, and two outputs. The input variables were: labour and deposit. The labour was measured by the personnel

costs, covered wages and all associated expenses expressed in thousands of EUR. The second input, deposit was measured by the total deposits received from clients and other credit institutions also expressed in thousands of EUR. As the output variables were determined: loan and net interest income. The output loan was measured as total loans to clients and other credit institutions expressed in thousands of EUR. The net interest income was expressed as the difference between interest incomes and interest expenses (in thousands of EUR). The reason for choosing interest income instead of net fee and commission income was that net interest income formed nearly 80% of bank's gross revenues in 2015, as Slovak and Czech banks still prefer interest rate policy before the fee policy.

For the profit-oriented intermediation approach (POIA) two inputs (two main types of costs) and two outputs (two main sources of revenues) were selected. The input variables were: interest and related expenses expressed in thousands of EUR and total operating expenses, calculated as a sum of personnel costs and other operating expenses (in thousands of EUR). As the output variables were used interest income expressed in thousands of EUR and non-interest income in thousands of EUR.

For both approaches, the data were extracted from banks' end-of-year unconsolidated balance sheets and income statements based on international accounting standards. All data were reported in EUR as the reference currency. The data in national currency (Česká koruna – CZK), were converted by using the official exchange rate of the Czech National Bank from 31.12.2015. Descriptive statistic of all input and output variables used in the analysis is given in table (Tab. I). As can be seen the Czech banking sector had higher volatility and can be considered bigger than the Slovak one.

I: Input and output variables in 2015 (in thousands of EUR)

		Personnel expenses	Total deposits	Total loans	Net interest income	Total operating expenses	Interest expenses	Interest income	Non-interest income
Czech Republic	Minimum	1,431	489,645	564,548	14,611	13,408	3,787	18,398	3,697
	Maximum	325,752	29,517,173	19,703,646	959,571	687,691	272,620	1,074,642	498,397
	Average	89,065	9,195,569	6,773,919	286,839	200,280	62,005	348,334	146,439
	St. deviation	103,897	10,280,770	7,536,261	329,427	218,643	77,002	389,987	166,779
Slovak Republic	Minimum	6,324	513,400	258,147	14,177	11,720	7,459	21,636	7,828
	Maximum	117,080	11,062,984	9,536,299	469,081	268,760	69,021	525,696	154,958
	Average	53,363	4,742,256	4,178,370	178,675	120,299	33,644	212,319	68,889
	St. deviation	44,006	3,980,229	3,655,247	159,965	99,439	20,250	177,965	60,337
Total	Minimum	1,431	489,645	258,147	14,177	11,720	3,787	18,398	3,697
	Maximum	325,752	29,517,173	19,703,646	959,571	687,691	272,620	1,074,642	498,397
	Average	74,459	7,373,759	5,712,103	242,590	167,561	50,403	292,691	114,714
	St. deviation	85,023	8,453,060	6,265,010	273,357	180,843	61,222	321,961	137,129

Source: Authors' calculations

1 <http://www.saitech-inc.com/products/prod-dsp.asp>

II: Average super-efficiency scores in 2015

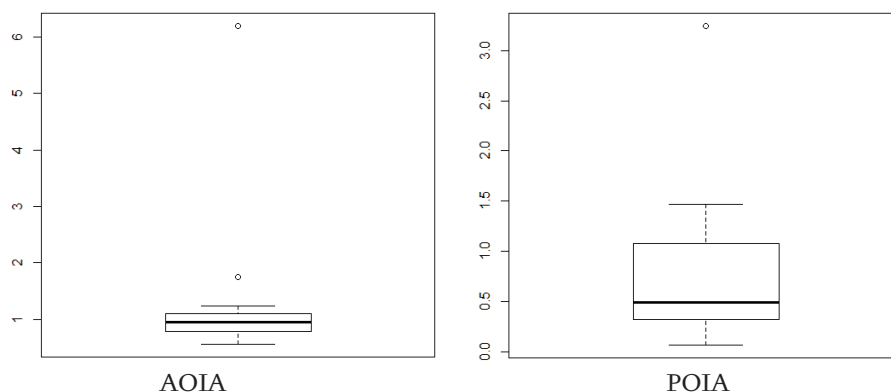
		Asset-oriented intermediation approach	Profit-oriented intermediation approach
Czech Republic	Minimum	0.5619	0.0685
	Maximum	6.1928	3.2432
	Average	1.3678	0.8700
	St. deviation	1.4827	0.8291
	No. of banks	13	13
	No. of super-efficient banks	6	6
Slovak Republic	Minimum	0.5853	0.2726
	Maximum	1.1626	1.4696
	Average	0.9154	0.6393
	St. deviation	0.1787	0.4482
	No. of banks	9	9
	No. of super-efficient banks	4	3
Total	Minimum	0.5619	0.0685
	Maximum	6.1928	3.2432
	Average	1.1827	0.7756
	St. deviation	1.1490	0.6949
	No. of banks	22	22
	No. of super-efficient banks	10	9

Source: Authors' calculations

Following the described methodology, we evaluated non-oriented non-radial super-efficiency (based on SBM model) of all banks in the estimation set under the assumption of a variable return to scale. We pooled the cross-country data and used them to define a common best-practice efficiency frontier. This allowed us to focus on determining relative differences in efficiency across banks. The estimated efficiencies were used to calculate average values separately on the “national” and “international” level. At the “national” level the average efficiencies were calculated as the simple arithmetic average of estimated efficiencies of Czech banks and Slovak banks separately. At the “international” level, the average efficiency was calculated from data of all banks.

The “international” and “national” average efficiencies were calculated in case of asset-oriented intermediation approach (AOIA) and profit-oriented intermediation approach (POIA). The results are recorded in table (Tab. II).

Under the asset-oriented intermediation approach, the average super-efficiency in the whole sample was 118.27%, there were 10 super-efficient banks and the super-efficiency of individual banks reached values from 56.19% to 619.28%. When we look at banking sectors separately, we can see that in the case of Slovak banks the level of super-efficiency was lower than in a case of Czech banks. Also, the number of super-efficient banks was lower in a case of Slovakia (SR 4 banks, CR 6 banks). The average super-efficiency in Czech



3: Boxplot of super-efficiency scores under the asset-oriented intermediation approach (AOIA) and profit-oriented intermediation approach (POIA) in 2015

Source: Authors' calculations

banking sector was 136.78%. In the case of Czech banks, there was reached minimum and maximum valid for the whole sample. In Slovak banking sector the average super-efficiency moved from 58.53% to 116.26%, where the average super-efficiency was 91.54%.

Under the profit-oriented intermediation approach, the average super-efficiency in the whole sample was 77.56%, there were 10 super-efficient banks and the super-efficiency of individual banks moved from 6.85% to 324.32%. When we look at banking sectors separately, we can see that in Slovakia the level of super-efficiency and the number of super-efficient banks was lower than in Czech Republic (SR 3 banks, CR 6 banks). The average super-efficiency in Czech banking sector was 87%. In the case of Czech banks, there was reached minimum and maximum valid for the whole sample. In Slovak banking sector the average super-efficiency moved from 27.26% to 146.96%, where the average super-efficiency was 63.93%.

In next part of our analysis, we try to compare our results through the boxplot analysis. As can be seen in the case of profit-oriented intermediation approach, the efficiencies were skewed towards lower values, which reflected by moving the median (horizontal line in the rectangle a restrictive value of 25<sup>th</sup> percentile and 75<sup>th</sup> percentile) down. The gap between the median and the 25<sup>th</sup> percentile was smaller than the gap between the median and the 75<sup>th</sup> percentile. According to asset-oriented intermediation approach, the values weren't skewed, as the differences between median and 25<sup>th</sup> percentile and median and 75<sup>th</sup> percentile were approximately the same. One of the advantages of the super-efficiency model is that we can identify the outliers which can deform the efficiency frontier and therefore they should be excluded from the analysis. These outliers can be also seen in next figure (Fig. 3). According to boxplot analysis two outliers can be seen under the AOIA (Fio banka, a.s., Expobank CZ, a.s.), and one outlier under the POIA (Moneta Money Bank, a.s.). The presence of outliers

### III: Super-efficiency scores and ranking of individual banks under the AOIA in 2015

No.	State	Name	AOIA		AOIA_without outlier		AOIA_change	
			Score	Ranking	Score	Ranking	Score (%)	Ranking (count)
1	SR	Československá obchodná banka, a.s.	0.74093	18	0.74093	16	0.00	2
2	SR	OTP Banka Slovensko, a.s.	0.83700	16	1.08325	6	29.42	10
3	SR	Poštová banka, a.s.	1.16261	4	1.16434	2	0.15	2
4	SR	Prima banka Slovensko, a.s.	0.99003	11	1.08525	5	9.62	6
5	SR	Privatbanka, a.s.	1.00000	10	1.00000	12	<b>0.00</b>	<b>-2</b>
6	SR	Sberbank Slovensko, a.s.	0.58531	21	0.65014	19	11.08	2
7	SR	Slovenská sporiteľňa, a.s.	1.03346	9	1.03346	11	<b>0.00</b>	<b>-2</b>
8	SR	Tatra banka, a.s.	0.84336	15	0.84336	14	0.00	1
9	SR	Všeobecná úverová banka, a.s.	1.04548	8	1.04578	9	<b>0.03</b>	<b>-1</b>
10	CR	Air Bank, a.s.	0.87414	14	1.03816	10	18.76	4
11	CR	Česká spořitelna, a.s.	1.08220	7	1.08220	7	0.00	0
12	CR	Československá obchodní banka, a.s.	1.10083	6	1.10083	4	0.00	2
13	CR	<b>Equa bank, a.s.</b>	<b>0.60681</b>	<b>20</b>	<b>0.70418</b>	<b>17</b>	<b>16.05</b>	<b>3</b>
14	CR	<b>Expobank CZ, a.s.</b>	<b>1.75258</b>	<b>2</b>				
15	CR	Fio banka, a.s.	6.19278	1				
16	CR	J&T Banka, a.s.	0.69451	19	0.69451	18	0.00	1
17	CR	Komerční banka, a.s.	0.89028	13	0.89028	13	0.00	0
18	CR	Moneta Money Bank, a.s.	1.23014	3	1.23014	1	0.00	2
19	CR	PPF banka, a.s.	0.89465	12	1.07614	8	20.29	4
20	CR	Raiffeisenbank, a.s.	0.78568	17	0.78568	15	0.00	2
21	CR	Sberbank CZ, a.s.	0.56186	22	0.59484	20	5.87	2
22	CR	UniCredit Bank Czech Republic and Slovakia, a.s.	1.11494	5	1.12163	3	0.60	2
		Minimum	0.56186		0.59484			
		Maximum	6.19278		1.23014			
		Average	1.18271		0.94826			
		St. deviation	1.14901		0.19192			

Source: Authors' calculations



in efficiencies could be caused by the fact that some banks were able to use their inputs to produce their outputs more effective compared to other banks in the sample. For example under the POIA the Moneta Money Bank, a.s. compared with Raiffeisenbank, a.s. was able to reach the interest income higher by 27% and non-interest income higher by 164% under the comparable level of total operating cost. Or compared with Tatra banka, a.s. was able to use lower costs (TOE lower by 17%, and IE lower by 76%) when producing approximately the same level of interest income.

The level of efficiency was also compared by the correlation analysis; the Pearson correlation coefficient was used. On the basis of results and according to the classification prepared by Cohen (1988), we can say, that between both approaches there existed a positive small correlation (0.2608). The results of correlation analysis between different approaches signalise, that if the bank was efficient in the process of transformation of labour and deposits to loans and net interest income it must not be

also efficient in the transformation of its cost to its revenues.

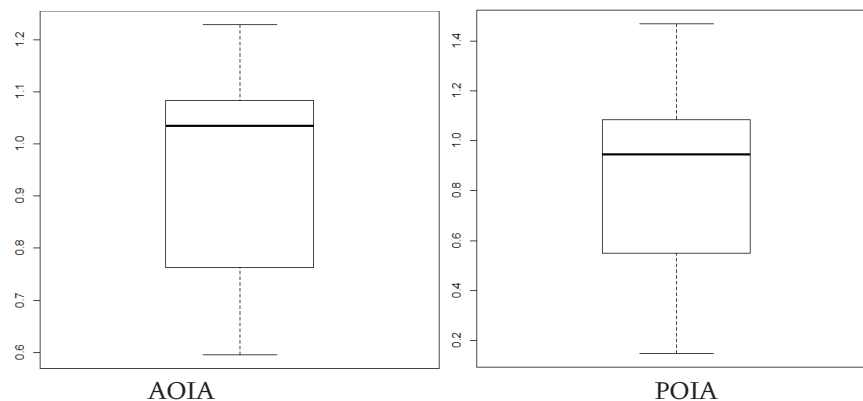
Tab. III and Tab. IV show information about the non-radial non-oriented super-efficiencies and ranking of individual banks in the Slovakia and Czech Republic in 2015. Based on the results we can say that transformation of total deposits and personnel costs into total loans and net-interest income was successfully achieved by ten banks of twenty-two banks. Under the POIA the transformation of total operating expenses and interest expenses into the interest income and non-interest income was successfully achieved by nine banks. The lowest level of efficiency within the group and under the used variables was reached under the AOIA by Sberbank CZ, a.s. (56.19%) from the Czech Republic, and under the POIA by Equa bank, a.s. (6.85%) from the Czech Republic.

We tried to compare the level of super-efficiency of individual banks after the exclusion of outliers from the dataset. As can be seen in table (Tab. III) the exclusion of outliers from the dataset led to

IV: Super-efficiency scores and ranking of individual banks under the POIA in 2015

No.	State	Name	POIA		POIA_without outlier		POIA_change	
			Score	Ranking	Score	Ranking	Score (%)	Ranking (count)
1	SR	Československá obchodná banka, a.s.	0.27263	19	0.64236	13	135.62	6
2	SR	OTP Banka Slovensko, a.s.	0.31787	17	0.56736	15	78.49	2
3	SR	Poštová banka, a.s.	1.13119	5	1.14005	5	0.78	0
4	SR	Prima banka Slovensko, a.s.	0.33658	16	0.58367	14	73.41	2
5	SR	Privatbanka, a.s.	1.46957	2	1.46957	1	0.00	1
6	SR	Sberbank Slovensko, a.s.	0.28658	18	0.49141	19	<b>71.47</b>	<b>-1</b>
7	SR	Slovenská sporiteľňa, a.s.	1.01894	9	1.06220	8	4.25	1
8	SR	Tatra banka, a.s.	0.37673	14	0.94440	11	150.68	3
9	SR	Všeobecná úverová banka, a.s.	0.54344	10	0.85687	12	<b>57.68</b>	<b>-2</b>
10	CR	Air Bank, a.s.	0.26093	20	0.54052	17	107.15	3
11	CR	Česká spořitelna, a.s.	1.25386	4	1.29037	3	2.91	1
12	CR	Československá obchodní banka, a.s.	1.03574	8	1.03574	9	<b>0.00</b>	<b>-1</b>
13	CR	Equa bank, a.s.	0.06852	22	0.14823	21	116.33	1
14	CR	Expobank CZ, a.s.	0.50536	11	0.54937	16	<b>8.71</b>	<b>-5</b>
15	CR	Fio banka, a.s.	1.30160	3	1.30160	2	0.00	1
16	CR	J&T Banka, a.s.	0.41674	13	0.53857	18	<b>29.23</b>	<b>-5</b>
17	CR	Komerční banka, a.s.	1.07544	7	1.08120	7	0.54	0
18	<b>CR</b>	<b>Moneta Money Bank, a.s.</b>	<b>3.24317</b>	<b>1</b>				
19	CR	PPF banka, a.s.	1.07617	6	1.08409	6	0.74	0
20	CR	Raiffeisenbank, a.s.	0.37406	15	1.24192	4	232.01	11
21	CR	Sberbank CZ, a.s.	0.22378	21	0.44310	20	98.01	1
22	CR	UniCredit Bank Czech Republic and Slovakia, a.s.	0.47436	12	1.01978	10	114.98	2
		Minimum	0.06852		0.14823		116.33	
		Maximum	3.24317		1.46957		-54.69	
		Average	0.77560		0.85868		10.71	
		St. deviation	0.69488		0.35406			

Source: Authors' calculations



4: Boxplot of super-efficiency scores under the asset-oriented intermediation approach (AOIA) and profit-oriented intermediation approach (POIA) without outliers  
Source: Authors' calculations

the better statistical characteristic of estimated efficiency. Under the AOIA the exclusion of both outliers led to the decrease of average efficiency of 19.82% and the variability significantly decreased. The minimal efficiency increased from 56.19% to 59.84% in case of Sberbank CZ, a.s. (CR). After shutting outliers the first place was occupied by the Moneta Money Bank, a.s. (CR), where the exclusion of outliers didn't get to change in super-efficiency. In the case of three banks can be seen the decrease in position, but it was connected with zero change in super-efficiencies. In a case of two banks there was any change in ranking and in a case of fifteen banks, there can be seen the improvement in ranking.

In the case of POIA, the exclusion of Moneta Money Bank, a.s. from the dataset led to the increase of average efficiency of 10.71% (see Tab. IV). The minimal efficiency increased from 6.85% to 14.82% in case of Equa bank, a.s. (CR). After shutting outlier the first place was occupied by the Privatbanka, a.s. (SR), where the exclusion of outlier didn't get to change in super-efficiency. In the case of five banks can be seen the decrease in position, in a case of two banks there were any change and in a case of ten banks, there can be seen the improvement in ranking. From the descriptive statistic of the sample can be also seen that exclusion led to the lower variability in efficiencies, which is represented by a lower value of standard deviation.

In the next part of our paper we compared the statistical characteristic of estimated efficiencies after exclusion of outliers through the boxplot and correlation analysis. Following figure (Fig. 4) displays the distribution of the efficiencies obtained

in the re-estimation process. As can be seen in both approaches there are not any other outliers. In case of both approaches the efficiencies were skewed towards higher values, which reflected by moving the median up, closer to the 75th percentile. The correlation coefficient between both approaches increased significantly from 0.2608 to 0.5267, which can be marked as strong correlation. This confirmed our hypothesis that exclusion of outliers led to the better statistical characteristic of estimated dataset and helps in calculating correlation coefficients.

Last mentioned, but not least advantage of DEA models is, that they bring recommendations for inefficient units how to change inputs or outputs to reach the efficiency frontier. In our analysis was used the non-oriented model, that captures the desire to improve both the inputs and outputs simultaneously. Tab. V summarises the findings in this area.

In general, we can say that for the movement to efficiency frontier under the asset-oriented intermediation approach it is necessary to increase personnel cost by 1.71%, total deposits by 0.27%, and net interest income by 10.78%, and to decrease total loans by 0.23% in average. Under the profit-oriented intermediation approach, it is necessary to increase total operating expenses by 1.91% and non-interest income by 4.55%, and to decrease interest expenses by 9.57%, and interest income by 0.87% in average. In general, we can say that for movement to efficiency frontier higher level of changes is necessary in the case of profit-oriented intermediation approach, what is also evident by the lower average value of POIA super-efficiency score.

## CONCLUSION

This study employed the non-radial and non-oriented super-efficiency model using slacks-based measure under the assumption of a variable return to scale to analyse the performance of twenty-two Czech and Slovak domestic commercial banks in 2015. The two main orientations of the intermediation approach (the asset-oriented and profit-oriented) were used. The banks were ranked according to both approaches. Under the asset-oriented approach, the average super-efficiency moved from 56.19% to 619.28% and there were 10 super-efficient banks. Under

V: Comparison of original and projection average values of input and output variables under different approaches (without outliers)

Asset-oriented intermediation approach												
Personnel expenses			Total deposits			Total loans			Net interest income			
	Data	Projection	Diff. (%)	Data	Projection	Diff. (%)	Data	Projection	Diff. (%)	Data	Projection	Diff. (%)
Average	81,397	82,790	1.71	7,983,103	8,004,963	0.27	6,217,929	6,203,464	-0.23	265,226	293,823	10.78
Max	325,752	345,187	14.15	29,517,173	29,517,173	35.17	19,703,646	19,703,464	0	959,571	959,571	136.23
Min	6,324	6,324	-7.05	513,400	513,400	-14.69	258,147	50,535	-80.42	14,177	9,767	-31.11
St. Dev.	86,204	89,315	4.26	8,637,745	8,596,226	8.75	6,357,558	6,371,502	17.98	276,869	277,724	45.45
Profit-oriented intermediation approach												
Total operating expenses			Interest expenses			Interest income			Non-interest income			
	Data	Projection	Diff. (%)	Data	Projection	Diff. (%)	Data	Projection	Diff. (%)	Data	Projection	Diff. (%)
Average	166,531	169,717	1.91	52,424	47,406	-9.57	291,727	289,202	-0.87	105,646	110,449	4.55
Max	687,691	687,691	15.02	272,620	272,620	96.98	1,074,642	1,074,642	45.67	498,397	498,397	818.40
Min	11,720	13,125	-11.67	3,787	6,485	-59.07	18,398	15,159	-23.11	3,697	7,828	-35.05
St. Dev.	185,242	190,976	5.70	61,977	64,018	36.32	329,879	327,724	12.55	133,585	127,068	178.54

\* Original data and projection are in thousands of EUR

Source: Authors' calculations

the profit-oriented approach, the super-efficiency of individual banks reached values from 6.85% to 324.32% and there were 10 super-efficient banks. We found out that in the case of Slovak banks the level of average super-efficiency was lower than in a case of Czech banks in both approaches. One of the advantages of DEA is, that it could be used to determine outliers. The boxplot analysis was used to present the outliers in the dataset. In the presented boxplot could be seen 3 outliers: one of them from Slovakia and two from Czech Republic. As the outlier can affect the estimation of efficiency frontier we tried to identify them and the production frontier was re-estimated using the remaining units. The result suggested that the exclusion of outliers from the dataset led to the better statistical characteristic of estimated efficiencies. Pointing out the use of super-efficiency model to find outliers can be considered as the main contribution of this paper. The identification of outliers and their removal are very important when we would like to use information about the efficiency of units in next research, for example in regression analysis. Outliers are important in regression models, where they can have a large influence on the estimates. According to Bogetoft and Otto (2011), we are particular concerned with units for which a variable is extremely large, meaning that the unit has potential leverage in influencing the shape and slope of the regression and that the unit is off-centre in the sense that they actually exercise their leverage.

For the future research, we can try to analyse the super-efficiency during the longer period and try to analyse what was the reasons for movements in super-efficiency frontier by using Malmquist index. As we can apply information about outliers gained by super-efficiency model we can also analyse the determinants which influence the development of efficiency in Czech and Slovak banking sector.

#### Acknowledgement

Research behind this paper was supported by the Czech Science Foundation within the project GAČR 16-17796S, and is in line with problems studied within the research project VEGA 1/0446/15 supported by the Slovak Scientific Grant Agency.

#### REFERENCES

- AHN, H. and LE, M. H. 2014. An insight into the specification of the input-output set for DEA-based bank efficiency measurement. *Management Review Quarterly*, 64(1): 3–37.
- BANKER, R. D. and CHANG, H. 2006. The super-efficiency procedure for outlier identification, not for ranking efficient units. *European Journal of Operational Research*, 175(2): 1311–1320.
- BENSTON, G. 1965. Branch banking and economies of scale. *Journal of Finance*, 20(2): 312–331.
- BANKER, R. D. and GIFFORD, J. L. 1988. *A relative efficiency model for the evaluation of public health nurse productivity*. Carnegie: Mellon University Mimeo.
- BERGER, A. N. and HUMPHREY, D. 1997. Efficiency of financial institutions: international survey and directions for future research. *European Journal of Operational Research*, 98(2): 175–212.
- BERGER, A. N. and MESTER, L. 2003. Explaining the dramatic changes in performance of US banks: technological change, deregulation, and dynamic changes in competition. *Journal of Financial Intermediation*, 12(1): 57–95.

- BOĎA, M. and ZIMKOVÁ, E. 2015. Efficiency in the Slovak banking industry: a comparison of three approaches. *Prague Economic Papers*, 24(4): 434–451.
- BOGETOFT, P. and OTTO, L. 2011. *Benchmarking with DEA, SFA, and R*. New York: Springer.
- COOPER, W., SEIFORD, L. M. and TONE, K. 2007. *Data Envelopment Analysis: A comprehensive text with models applications*. New York: Springer Science.
- CESIFO. ©2014. *Database for Institutional Comparisons in Europe. Business and Financial Markets*. [Online]. Available at: <http://www.cesifo-group.de/ifoHome/facts/DICE/Business-and-Financial-Markets.html> [Accessed: 2016, July 11.].
- IRŠOVÁ, Z. and HAVRÁNEK, T. 2011. Bank efficiency in transitional countries: Sensitivity to stochastic frontier design. *Transition Studies Review*, 18(2): 230–270.
- KASEM, E. et al. 2015. Key sustainability performance indicator analysis for Czech breweries. *Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis*, 63(6): 1937–1944.
- KOČIŠOVÁ, K. 2014. Application of Data Envelopment Analysis to measure cost, revenue and profit efficiency. *Statistika*, 94(3): 47–57.
- MISHKIN, F. S. 2009. *The Economics of Money, Banking, and Financial Markets*. USA: Pearson education.
- MYŠKOVÁ, K. and ŽÁK, J. 2016. Data Envelopment Analysis for technological, environmental and economic analysis of motorway underpasses. *Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis*, 64(1): 307–314.
- PALEČKOVÁ, I. 2015. Estimation of the efficiency of Slovak commercial banks by the Data Envelopment Analysis [In Czech: Odhad efektivnosti slovenských komerčních bank použitím Data Envelopment Analysis]. *Acta Academica Karvinensia*, 2015(1): 130–140.
- PANČUROVÁ, D. and LYÓCSA, Š. 2013. Determinants of commercial bank's efficiency: Evidence from 11 CEE countries. *Finance a úvěr: Czech Journal of Economics and Finance*, 63(2): 152–179.
- POLOUČEK, S. et al. 2004. *Reforming the financial sector in Central European countries*. Hampshire: Palgrave Macmillan Publishers.
- SEALEY, C. W. and LINDLEY, J. T. 1977. Inputs, outputs, and a theory of production and cost at depository financial institutions. *The Journal of Finance*, 32(4): 1251–1266.
- SENDEK, S., SVITÁLKOVÁ, Z. and ANGELOVIČOVÁ, K. 2015. Efficiency evaluation of hospitals in the environment of the Czech and Slovak Republic. *Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis*, 63(6): 2109–2118.
- SHERMAN, H. D. and GOLD, F. 1985. Bank branch operating efficiency: Evaluation with Data Envelopment Analysis. *Journal of Banking and Finance*, 9(2): 297–315.
- STAVÁREK, D. 2005. *Restructuring of the banking sectors and bank efficiency in Visegrad countries* [In Czech: Restrukturalizace bankovních sektorů a efektivnost bank v zemích Visegrádské skupiny]. Karviná: Silesian University, School of Business Administration.
- STAVÁREK, D. and ŘEPKOVÁ, I. 2012. Efficiency in the Czech banking industry: A non-parametric approach. *Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis*, 60(2): 357–366.
- STEFKO, R., GAVUROVÁ, B. and KORÓNY, S. 2016. Efficiency measurement in healthcare work management using Malmquist indices. *Polish Journal of Management Studies*, 3(1): 168–180.
- SVITÁLKOVÁ, Z. 2014. Comparison and evaluation of bank efficiency in selected countries in EU. *Procedia Economics and Finance*, 12: 644–653.
- TIMMER, C. P. 1971. Using a probabilistic frontier production function to measure technical efficiency. *Journal of Political Economy*, 79(4): 776–794.
- VAVŘINA, J., HAMPEL, D. and JANOVÁ, J. 2013. New approaches for the financial distress classification in agribusiness. *Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis*, 61(4): 1177–1182.
- ZIMKOVÁ, E. 2014. Technical efficiency and super-efficiency of the banking sector in Slovakia. *Procedia Economics and Finance*, 12: 780–787.
- ZIMKOVÁ, E. 2015. Technical efficiency and super-efficiency of the insurance sector in Slovakia. *Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis*, 63(6): 2205–2211.

#### Contact information

Assoc. prof. Kristína Kočíšová, PhD.: [kristina.kocisova@tuke.sk](mailto:kristina.kocisova@tuke.sk)  
 Ing. Iveta Palečková, PhD.: [paleckova@opf.slu.cz](mailto:paleckova@opf.slu.cz)