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HYBRID ARIMA AND SUPPORT VECTOR REGRESSION IN SHORT-TERM ELECTRICITY PRICE FORECASTING

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Abstract

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The literature suggests that, in short-term electricity-price forecasting, a combination of ARIMA and support vector regression (SVR) yields performance improvement over separate use of each method. The objective of the research is to investigate the circumstances under which these hybrid models are superior for day-ahead hourly price forecasting. Analysis of the Nord Pool market with 16 interconnected areas and 6 investigated monthly periods allows not only for a considerable level of generalizability but also for assessment of the effect of transmission congestion since this causes differences in prices between the Nord Pool areas. The paper finds that SVR, SVRARIMA and ARIMASVR provide similar performance, at the same time, hybrid methods outperform single models in terms of RMSE in 98 % of investigated time series. Furthermore, it seems that higher flexibility of hybrid models improves modeling of price spikes at a slight cost of imprecision during steady periods. Lastly, superiority of hybrid models is pronounced under transmission congestions, measured as first and second moments of the electricity price.

Keywords: short-term electricity price forecasting, hybrid models, time series, ARIMA models, support vector regression, transmission congestion, Nord Pool electricity market

INTRODUCTION

Electricity market deregulations in 1990s caused a need for electricity price forecasting (EPF) due to leaving electricity price determined by supply and demand, which resulted in substantial volatility of electricity price and thus elaborate predictions of future price became necessary for agents on the market. Next-day prices are important to forecast for mainly three groups of agents: electricity producers, retailers and large industrial firms. For example, if a firm contracts a surplus of electricity, it is obliged to pay not only for the ordered amount but also for the surplus, hence paying for unused energy twice. On the other hand, shortage of electricity causes underproduction and loss of profit. Therefore, the costs of imprecise amount of contracted electricity cause significant losses and can lead to bankruptcy (Weron, 2014).

Non-storability of electricity causes failure of the no-arbitrage condition known from most markets. Even if the agents know that future price will increase, the electricity cannot be bought, stored and resold in the future for a higher price. Thus, the electricity price can be relatively precisely forecasted since its changes correspond mainly with changes in demand which are to some extent predictable. For instance, electricity demand usually peaks around noon and is lowest during early morning hours, or is generally lower during weekends. The seasonal behavior of demand determines electricity price seasonality. Considering the time dimension, differences in price are based on the daily hour, the weekday, the month and the year. Specificity of electricity price behavior affects the choice of suitable forecasting techniques since the electricity price series is subject to seasonality, pronounced volatility, positive skewness, excess kurtosis, and conditional heteroscedasticity (Karakatsani and Bunn, 2008).

Forecasting techniques can be perceived according to their flexibility, i.e., capability to capture irregular and volatile behavior. Flexible techniques are usually more complex hence require more observations, at the same time, decent performance is often achieved in modeling volatile time series. Rather nonflexible approaches are time series methods, which are usually parametric. Parametric models assume some distribution of price shocks, whereas, nonparametric models leave the price series unrestricted. The most-used and oldest models in EPF are autoregressive (AR) processes, which are parametric. AR models often incorporate a moving-average (MA) part, sometimes consider additional variables (Weron and Misiorek, 2005), and usually perform sufficiently during steady periods, whereas, price spikes and periods of substantial volatility are beyond their capabilities (Weron, 2014). More flexible methods are usually nonparametric, with machine learning (computer intelligence) approaches being the most successful (Weron, 2014). Machine learning approaches are often capable of capturing substantial non-linearity, thus, can be seen complementary to ARMA-type models, which is the perspective adopted in this paper. Disadvantage of machine learning techniques is their inability to interpret the model since the only output is usually the prediction.

By combining an ARMA-type model with a machine learning technique, namely support vector regression (SVR), flexibility of this hybrid model should increase compared a single-method model. In addition, one can utilize advantages from each of the two worlds: attaining precise forecasts by capturing nonlinear behavior while maintaining some interpretability. This research aims to compare two specifications of hybrid models based on ARIMA combined with SVR since the SVR models have shown to yield high performance in this combination (Weron, 2014). Furthermore, by comparing multiple areas of the Nord Pool market, the effect of transmission congestion is assessed since insufficient network capacity induces differences in prices between areas (Kristiansen, 2014; Loland *et al.*, 2012). The effect of transmission congestion on forecasting precision is a part of balancing costs, i.e., enters the final electricity price.

The paper shows very similar performance of the two most-used sequential approaches to the ARIMA-SVR combination. The first approach estimates ARIMA and then predicts residuals by SVR, the second approach is vice versa, i.e., SVR predicting the price and ARIMA the residuals. The paper finds that, in terms of RMSE, performance of both hybrid methods is close to a single SVR method, nevertheless, the single SVR method is superior in only 2 % of the investigated times series. Furthermore, it seems that the value added of the hybrid methods lies in improved capturing of price spikes at a slight cost of increased imprecision during steady periods. Lastly, volatility and transmission constraints seem to negatively affect predictability of day-ahead prices, which can be perceived as a cost for agents operating at an electricity market.

Literature Review

Agents in the market have different information on which they can base their estimates of future prices. For example, dominant producers or integrated companies possess critical information on market fundamentals hence create asymmetries across agents (Karakatsani and Bunn, 2008). According to the dominance of agents, two streams of research are apparent in the literature. The prevalent approach for dominant agents is based on multi-agent models (Ventosa *et al.* 2005), which allow for incorporating private information in order to forecast price changes caused by shifts in supply or demand; thus, yielding rather qualitative than quantitative results (Weron, 2014). The approach for less-influential agents uses fundamental, reduced-form, statistical or machine learning methods (Weron, 2014) with usually publicly available information and produces estimation methods for agents, for which extra private information is not available. The latter is the subject of interest in this research.

For about last five years the EPF literature has been investigating hybrid methods based on support vector regression (SVR) algorithms (Chaabane, 2014; Che and Wang, 2010; Kavousi-Fard and Kavousi-Fard, 2013; Saleh *et al.*, 2014). These studies report substantial success of these hybrid approaches compared to using a single method. Of all flexible methods, SVR are preferred due to their out-of-sample performance, avoidance of local optima problems (Basak *et al.*, 2007) and the ability to visit infinite-dimensional space due to kernels (Christmann and Steinwart, 2007). Saleh *et al.* (2014) suggest that SVR offers more precise electricity price predictions than artificial neural networks, which have been popular in EPF (Weron, 2014).

Khashei and Bijari (2010) provide the motivation for using a combination of methods: one either cannot identify the data-generating process, or one is unable to capture all characteristics of the time series using a single method. Thus, the combination provides more scope for capturing the characteristics of the process. In the case of SVR hybrid methods, the electricity price is usually considered to be composed of two parts: linear and non-linear. The non-linear part is explained by SVR, while the linear part is described by some ARMA-type process. ARMA-type models have been the workhorse of EPF since its beginning, and even currently their performance is reasonably useful (Weron, 2014). Cuaresma *et al.* (2004) have shown that various extensions of ARMA models bring substantial improvements in terms of forecasting precision.

The current EPF literature on ARMA-SVR hybrid methods seems not to investigate generalizability since usually a few models are estimated for one or two periods. The need for many periods should increase with the variety of patterns and their length, both of which seem to be high in European electricity markets that are usually under study (Bosco *et al.*, 2010). Considering the ARMA-SVR studies available, Che and Wang (2010) have somewhat arbitrarily selected two weeks of the Californian electricity prices. In addition, both Chaabane (2014) and Che and Wang (2010) have used only 100 data points of the Nord Pool prices to evaluate the forecasting error. Both Saleh *et al.* (2014) and Kavousi-Fard and Kavousi-Fard (2013) have employed the hybrid approach to forecast 30 days of electricity load in Iran. Thus, generalization of the findings in the current literature appears troublesome. Furthermore, investigation of the effect of transmission congestion on day-ahead forecasting precision was not found in the literature.

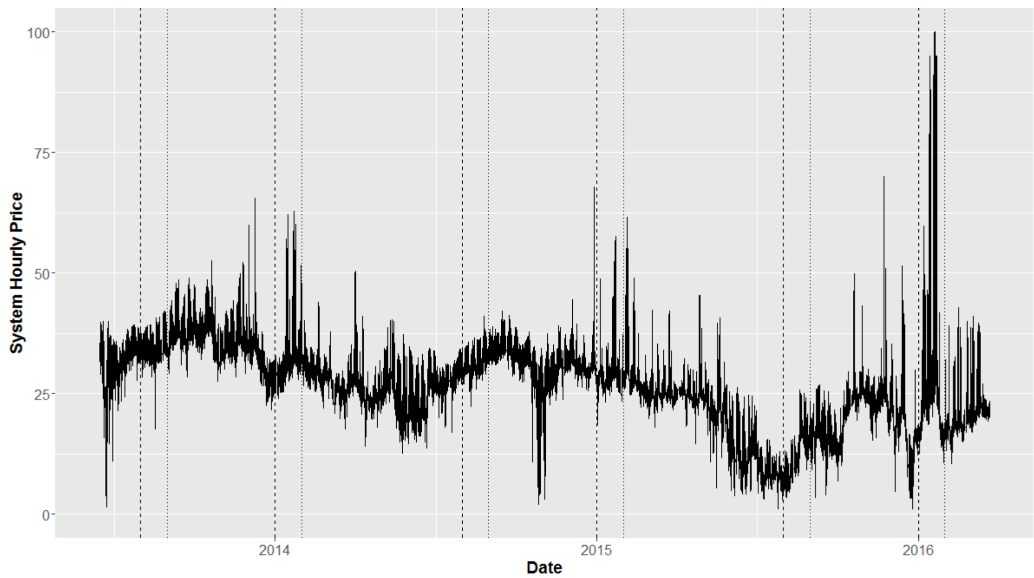
The Nordic Electricity Market

The Nordic Power Market (Nord Pool) was established as a consequence of power market deregulations of Nordic countries in the early 1990s. By the end of 2000, the market was operating for Norway, Sweden, Finland, and Denmark. In 2010-2013, Estonia, Latvia, and Lithuania joined. A substantial feature of the market is that about 50 % of electricity is generated by hydropower plants, which allows for some level of storage, thus giving a possibility to slightly smooth the prices. The effect is most pronounced in Norway where about 95 % of power is generated by hydro. Nevertheless, as depicted on Fig. 1, March 2013–March 2016, the hourly electricity price was subject to noticeable volatility. The system price is the price disregarding transmission constraints. Furthermore, Fig. 1 shows forecasted periods: August 2013, January 2014, August 2014, January 2015, August 2015, and January 2016. August prices do not have many large price spikes, whereas, January prices exhibit more volatile behavior, which is challenging to capture for any forecasting method. Therefore, variety of time series allowing some generalizability is captured.

For each of the 6 periods, hourly day-ahead prices are forecasted for each of the following 16 price areas: System (baseline price), Norway (6 areas), Sweden (4), Finland (1), Denmark (2), Estonia (1), Latvia (1). The areas serve for differentiation between prices due to transmission congestions. The level of congestion can be viewed in two ways: intensive and extensive. Both dimensions are captured by the mean of prices, whereas, extensiveness only can be computed as a percentage of time when the area price is higher than the system price. Furthermore, congestion not only causes the price of the area with high demand to increase but also causes the price of the area which would deliver some power under no limitation to decrease.

Tab. I captures descriptive statistics of price areas for Mar 2013–Mar 2016. Strong positive correlations are found between mean price and standard deviation (0.75), mean price and extensive congestion (0.84), standard deviation and extensive congestion (0.67), which suggests that transmission constraints not only increase average price but also volatility of prices. The causal channel hypothesized is based on the general ability of large grids to smooth demand and supply spikes in different parts of the grid. Thus, areas with insufficient connection should be more prone to variation due to local demand and supply shocks.

The main institution for trading electricity is the day-ahead market, therefore, forecasting in this paper obeys its information flows. Buyers and sellers submit bids consisting of price and quantity for each



1: *Nord Pool System Electricity Prices and Forecasted Periods.*
Source: the author

I: *Descriptive statistics of Nord Pool areas (Mar 2013–Mar 2016), price in EUR/MWh.*

Area	Bergen	Kr.sand	Oslo	SYS	Troms.	Molde	Tr.heim	SE1
Mean Price	25.6	25.6	25.8	27.2	27.5	28.0	28.0	28.0
St. Dev.	4.6	4.7	5.2	5.6	5.0	5.5	5.5	6.3
Time > SYS[1]	0.19	0.19	0.20	0.00	0.39	0.49	0.49	0.42
Area	**SE2**	**SE3**	**DK1**	**SE4**	**DK2**	**FI**	**EE**	**LT**
Mean Price	28.0	28.6	29.1	29.5	30.2	34.2	37.0	48.1
St. Dev.	6.3	7.9	21.3	9.3	11.3	13.6	16.1	19.9
Time>SYS	0.42	0.46	0.39	0.49	0.52	0.75	0.80	0.94

[1]Extensive congestion: Fraction of time the area price is higher than the system price, i.e., an extensive measure of congestion.
Source: the author.

hour of the next day. The market closes at 12:00 CET and according to the bids, equilibrium price and quantity are calculated. The next day, on hourly basis, contracts are physically delivered. Price areas serve for addressing bottlenecks in the system by increasing the price for area with relatively high demand and low supply and vice versa.

Forecasting methods

Five forecasting methods are implemented: naïve, ARIMA, SVR, ARIMASVR, SVRARIMA. In the literature of the hybrid setup with SVR, various time series specifications were identified: Che and Wang (2010), Karakatsani and Bunn (2008), Kavousi-Fard and Kavousi-Fard (2013), Pai and Lin (2005), and Saleh *et al.* (2014) use ARIMA, Yan and Chowdhury (2013) use ARMAX, Chaabane (2014) uses ARFIMA. This paper employs the most-common ARIMA approach. Hourly price data serve directly as an input for these methods and each method has been estimated for each of the 6 periods and for each of the 16 price areas, thus yielding 480 time series forecasts in total. Description of the implemented methods including their application in this paper is provided in the following sections: Naïve method; ARIMA; SVR; Hybridization: ARIMASVR, SVRARIMA.

Naïve method

The naïve method is often used as a benchmark method for calculation of error measures (e.g., Hyndman and Koehler, 2006) and in this paper serves the same purpose. The naïve approach forecasts hourly prices for the next day as prices of the current day. Any more sophisticated forecasting method should outperform this strategy. One can perceive the naïve method as a measure of forecasting difficulty between series since, e.g., more volatile and spiky series, which are generally more difficult to forecast, achieve worse score using this method.

ARIMA

In this paper, forecasting with ARIMA is based on the STL decomposition introduced by Cleveland *et al.* (1990), which divides the time series into a trend, a seasonal, and a remainder component using LOESS[1] smoothing. The trend and the seasonal component are subtracted from hourly prices, the remainder is forecasted by a non-seasonal ARIMA model. Then, forecasts are reseasonalized by adding the most recent forecast of the seasonal part, i.e., forecasting the seasonal part by the naïve approach. It is allowed for non-additive decompositions by using a Box-Cox transformation on the data before and after forecasts are computed. A non-seasonal ARIMA model can be written as:

$$x_t = \alpha + \sum_{j=1}^{p} \varnothing_j x_{t-j} + \sum_{k=1}^{q} \theta_k \varepsilon_{t-k} + \varepsilon_t$$

where x is a differenced time series and ε are errors. A triplet (p, d, q) denotes the order of AR(p) and MA(q) polynomial with the order of first differencing d. For determining the optimal value of (p, d, q), a routine by Hyndman and Khandakar (2007) using unit-root tests, MLE and minimization of the AIC criterion is implemented.

The STL decomposition and ARIMA(p, d, q) are estimated on two months of hourly prices and 24 hourly prices for the next day are computed. Then, the period of two months shifts by one day and the whole forecasting procedure is repeated. This relatively long period was chosen in order to allow for convergence of the algorithm.

SVR

A version of support vector machine for regression estimation (SVR) was developed by Vapnik, Golowich and Smola (1997). The underlying idea of SVR is to fit a linear regression function using a nonlinear mapping from a high-dimensional input space (Basak *et al.*, 2007), thus to overcome the problem of multidimensional function

1 LOESS refers to a nonparametric locally-weighted polynomial regression developed by Cleveland (1979).

estimation by reducing the number of estimated parameters (Vapnik *et al.*, 1997). The input space is transformed via kernels into a low-dimensional feature space and its parameters are estimated and used for forecasting, which provides also higher generalizability than forecasting using many parameters.

In this paper, estimation of SVR employs lagged values up to the 24^{th} order, which are daily-hour-specific, followed by lags of order 48 and 72. Each daily hour uses specific orders of lags according to the availability of information. For example, the hour after midnight uses all lags from the 1^{st} up to the 24^{th} order, whereas, 20:00–21:00 price exploits only lags of the 21^{st} up to the 24^{th} order. A month of hourly prices is used to train the model. The setup of the minimization problem is described in (Basak *et al.*, 2007), equation 15–18. A four-fold cross-validation is conducted in order to choose the best values for the following parameters: a cost parameter for misclassification C, a measure of precision ε, and a parameter γ of the radial-basis-function kernel, which allows mapping into higher-dimensional space. Similarly to ARIMA forecasting, the model is reestimated for each day with the exception that cross-validation search for hyperparameters is conducted only once for the time series.

Hybridization: ARIMASVR, SVRARIMA

Electricity price y can be generally viewed as a function of its linear component L and non-linear component N (Chaabane, 2014): $y = f$ (L,N). Usually, the relationship between L and N is considered additive (Kavousi-Fard and Kavousi-Fard, 2013; Pai and Lin, 2005). Two sequential approaches are investigated. A more common approach is to forecast price using ARIMA and then employ SVR to characterize non-linear behavior of ARIMA residuals (e.g., Chaabane, 2014; Kavousi-Fard and Kavousi-Fard, 2013; Pai and Lin, 2005; Saleh *et al.*, 2014), thus building the overall prediction as a sum of the predicted y by ARIMA and the predicted residual by SVR, i.e., ARIMASVR. A less-common approach is vice versa: using SVR first with ARIMA predicting residuals (Yan and Chowdhury, 2013), i.e., SVRARIMA. Che and Wang (2010) implement both approaches and report a slightly better performance of the latter.

Three months of data are used to obtain prediction errors of hybrid models. The first month serves for estimating the model forecasting price, subsequently, the price is forecasted for the second and third month. Then, residuals for these two months are computed, the other type of model is estimated on the second month, and residuals are forecasted for the third month. Forecasting follows the same procedure as with individual methods, which are described in section ARIMA and SVR. Predicted price is the sum of predicted price and predicted residuals for the third month, i.e., August or January.

Hypotheses

As the literature suggests, hybrid models should perform better than approaches based on a single method.

Hypothesis 1: Adding SVR to forecast ARIMA residuals does not provide a significant forecasting performance improvement.

Hypothesis 2: Adding ARIMA to forecast SVR residuals does not provide a significant forecasting performance improvement.

According to the outlined theory, transmission congestion increases both mean price and volatility. Enhancing ARIMA by SVR should provide more precise forecasts especially under high volatility and price spikes since ARIMA is not suitable for capturing irregular behavior. Employing the same logic, ARIMA modeling SVR residuals could contribute to forecasting precision due to its ability to capture linear behavior of time series. The argument for ARIMA benefitting to SVR model seems somewhat weaker since SVR is more flexible.

Hypothesis 3: ARIMA performance improvement by SVR predicting residuals does not increase with mean price.

Hypothesis 4: SVR performance improvement by ARIMA predicting residuals does not increase with mean price.

Hypothesis 5: ARIMA performance improvement by SVR predicting residuals does not increase with volatility of price.

Hypothesis 6: SVR performance improvement by ARIMA predicting residuals does not increase with volatility of price.

In order to test hypotheses 3–6, the relationship between performance improvement and measures of congestion is investigated. The following flexible specification provides a straightforward approach to investigate both linear and nonlinear relationships:

$$\begin{aligned} improvement &= \alpha + \beta_1 sd + \beta_2 sd^2 + \beta_3 sd^3 + \\ &+ \gamma_1 mean + \gamma_2 mean^2 + \gamma_3 mean^3 + \delta\left(sd \times mean\right) + \varepsilon \end{aligned} \tag{1}$$

where *improvement* refers to the difference between a hybrid and a non-hybrid model in terms of forecasting error, *sd* denotes standard deviation of price and *mean* is the average price over the forecasted period. Standard deviation is a measure of volatility, whereas, both standard deviation and mean are measures of congestion. β-coefficients are of interest when investigating hypotheses about volatility; γ-coefficients report the effect of mean price; β, γ, δ-coefficients serve for studying the effect of congestion including nonlinear relationships between the variables of interest. Inclusion of higher powers and interactions of explanatory variables covers possible interesting relationships between the performance improvement and congestion. The parameters are estimated by Ordinary Least Squares since this approach provides unbiased estimates of parameters under the assumption of exogeneity, which seems plausible in this scenario[2]. Model validation tests are conducted in the Results section.

Forecasting error evaluation

For each of the 6 periods, each of the 5 methods, and each of the 16 price areas, the error measures comparing the real price with the forecasted price were calculated as:

$$RMSE = \sqrt{\frac{1}{744}\sum_{t=1}^{744}\left(\widehat{price}_t - price_t\right)^2}$$

$$MAE = \frac{1}{744}\sum_{t=1}^{744}\left|\widehat{price}_t - price_t\right|$$

Both RMSE and MAE are among the most-common error measures for evaluation of point forecasts (Weron, 2014). Comparison of RMSE and MAE provides additional insights to the comparison of the investigated methods since RMSE emphasizes high residuals, which occur usually in the case of price spikes.

RESULTS

This section proceeds as follows: firstly, ARIMA specifications that were found most appropriate are described. Secondly, each method is evaluated and compared with others. Thirdly, the relationship between performance improvement caused by hybridization and transmission congestion is investigated. For the whole analysis, software R version 3.3.1 (R Core Team, 2016) was used. For the ARIMA modelling the R package *forecast* version 7 (Hyndman, 2016) and for SVR modelling the R package *e1071* version 1.6 were used (Meyer *et al.*, 2015).

The automated procedure for fitting ARIMA used first differences in 93 of 96 cases (6 periods of 16 price areas). The most-chosen orders (p, d, q) were (2,1,2) in 45 %, followed by (3,1,2) in 20 % and (3,1,3) in 15 % of cases. ARIMA modeling SVR residuals has orders (1,0,2) in 17 % and (1,0,1) in 14 % of cases and first differences are used only in 29 cases. Using the STL decomposition, while ARIMA(2,1,2) seems dominant in modeling prices, chosen orders for modeling residuals span from 0 to 5 for both AR and MA.

Fig. 2 shows RMSE for each period-area pair and captures summary statistics. Both of these show that, on average, all suggested methods outperform the naïve approach. Interestingly, in 11 % of cases, the naïve method yields the best performance. These are the cases when the price behavior substantially changes between the period used for estimation and the forecasted period. However, the naïve approach is subject to large residuals, thus, it cannot be recommended as the only forecasting method. In the case of RMSE, single ARIMA and SVR are almost never the best approaches, i.e., hybrid methods almost always improve the forecasting error. This result indicates that the investigated hybrid methods are beneficial especially for volatile series since RMSE overweights large residuals.

Considering MAE, Tab. II shows that the result is similar to RMSE with the exception that single SVR is the best model in 28 % of cases and on average reaches the lowest MAE. The single SVR is superior for slightly less-volatile time series, without statistical significance. This finding together with superiority of hybrids under RMSE supports the theory that a combination of methods, which provides additional flexibility, is beneficial especially for volatile series.

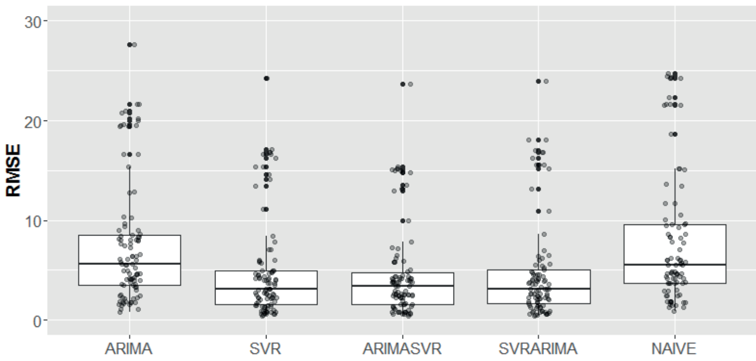
ARIMA yields relatively high error measures, whereas, all methods employing SVR perform on average similarly. ARIMASVR performs significantly better than ARIMA (t-test for the difference in means in terms of RMSE: p-value = 0.000 3). The value added of SVR compared to ARIMA is noticeable especially under high volatility. Fig. 3 shows that method using SVR can better accommodate large prices changes. Conversely, SVRARIMA does not provide significant improvement over SVR (p-value = 0.97). It seems that in order to achieve a decent forecasting performance, SVR should be included. Hypothesis 1 is thus rejected: SVR predicting ARIMA residuals significantly improves

II: *Summary statistics.*

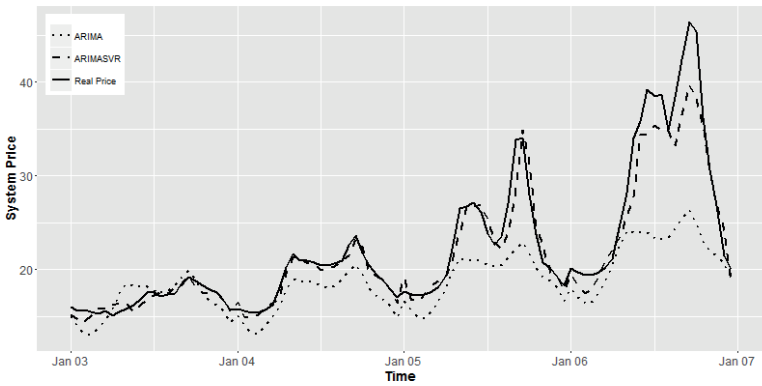
	Statistic	Naïve	ARIMA	SVR	ARIMASVR	SVRARIMA
RMSE	Mean	10.1	7.5	4.8	4.6	4.8
	St. D.	10.6	6.1	5.0	4.6	5.1
	Best method[1]	0.11	0.00	0.02	0.34	0.53
MAE	Mean	2.19	2.07	1.38	1.40	1.40
	St. D.	1.20	1.14	0.59	0.60	0.62
	Best method[1]	0.11	0.00	0.28	0.22	0.39

[1]Percentage of time this method provides the smallest error.
Source: the author.

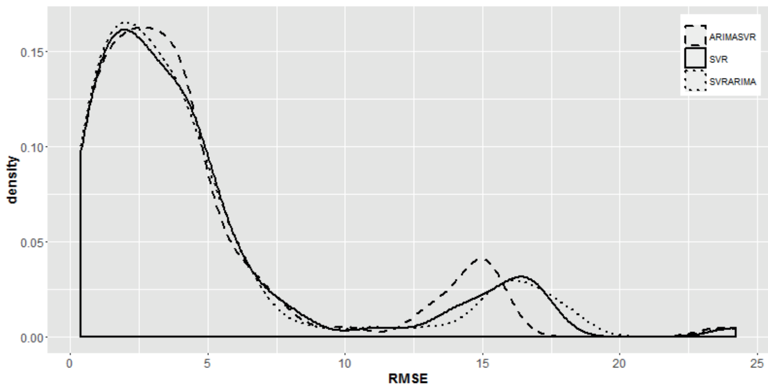
2 The electricity price distribution is obviously external to the improvement of forecasting precision, in addition, it is assumed that improvement in forecasting performance results from the changes in price distribution which is for the purpose of this exercise sufficiently described by mean, standard deviation and their simple transformations.



2: *Distributions of RMSE.*
Source: the author.



3: *ARIMA and ARIMASVR (Jan 2016).*
Source: the author.



4: *A comparison of hybrid methods.*
Source: the author.

forecasting performance in terms of RMSE. Hypothesis 2 is not rejected: ARIMA forecasting the SVR residuals does not provide a significant forecasting improvement. Considering RMSE, Tab. II shows that ARIMASVR yields on average the best performance, whereas, SVRARIMA is the best method in most cases. Nevertheless, differences between these two methods are not significant. Fig. 4 investigates the benefit of ARIMASVR over other methods using SVR. The largest value added seems to occur for RMSE about 15, i.e., for rather volatile time series.

In order to assess the effect of volatility and congestion on performance improvement of adding SVR modeling ARIMA residuals, equation 1 was estimated via OLS. Performance improvement is defined as a difference between the RMSE of ARIMA and the RMSE of ARIMASVR for each period-area pair.

Tab. III section ARIMA vs. ARIMASVR shows the OLS output with robust standard errors, which overcome any possible issues of heteroscedasticity. The Ramsey's RESET test suggests no misspecification of the functional form (p-value = 0.68). Furthermore, adjusted $R^2 = 0.81$ indicates that 81 % of the variation of performance improvement can be explained by transmission congestion variability, thus providing additional evidence of model validity.

Standard deviation and mean are significantly linked with forecasting error improvement. Using MAE yields significant (p-value = 0.04) and positive coefficient for sd, while the effect of mean is insignificant. Adding RMSE of the naïve method as a proxy for forecasting difficulty is insignificant. Positive coefficients indicate higher benefit of using SVR predicting ARIMA residuals compared to using only ARIMA when mean or variance increases. Holding the mean price fixed, a higher volatility of the time series by one standard deviation is related to an increase of the difference between RMSE of ARIMA and ARIMASVR by 1.55. Holding the volatility fixed, the effect of mean is 1.05. With respect to the size of coefficients, both effects seem to be approximately linear. Excluding sd^2 and sd^3 from the specification does not substantially change the result: parameter for sd decreases to 1.13 while the parameter for the mean increases to 1.30.

Using performance improvement of SVRARIMA over SVR as a dependent variable in equation 1 yields insignificant effects of sd and mean, which seems to correspond with very similar performance of these two methods. The results are provided in Tab. III: SVR vs. SVRARIMA.

Evidence shown in Tab. III leads to rejection of hypotheses 3 and 5 about sufficiency of ARIMA modelling in comparison with ARIMASVR, whereas, hypotheses 4 and 6 about sufficiency of SVR modelling in comparison with SVRARIMA are not rejected. It is concluded that benefit of ARIMASVR over ARIMA seems to increase with transmission congestion.

DISCUSSION

Under RMSE, the hybrid methods provide better forecasting results than their single counterparts in 98 % of the cases. The remaining 2 % are represented by SVR. This result of superiority of hybrid methods is similar to the findings of the current literature. Investigation of the defined hypotheses reveals the insufficiency of ARIMA as the only forecasting method since SVR provides superior results and capability of modelling the ARIMA residuals. Moreover, the value added of combining ARIMA with SVR seems to increase for more volatile time series and for areas that are subject to substantial transmission congestions. Under MAE, the single SVR seems to be the best approach, which together with the RMSE results indicates that additional flexibility of hybrid methods improves capturing of price spikes at a slight cost of imprecision during steady periods.

A coarse benefit analysis of no transmission congestion in terms of forecasting precision is conducted. The average MAE of ARIMASVR is 1.04 under no transmission congestion, whereas, for all price areas amounts to 1.40. Thus, on average, the hourly price would be predicted more precisely by €0.36 under no transmission congestion. The costs of imprecise forecasting are part of balance costs, which increase the final electricity price due to higher uncertainty.

The absence of transmission constraints would cause some volatility and higher prices in the most stable and cheapest areas. In the data, two Norway areas are more precisely predicted than the System price: Bergen and Kristiansand. These two areas are also the only areas with mean price lower than the System price. Thus, arguably, 13 areas would be better-off and 2 areas would be worse-off in the absence of transmission congestions.

Two limitations of the analysis are worth noticing. Firstly, this study is limited to ARIMA-SVR specifications; therefore, it cannot be claimed

III: *OLS output of equation 1, robust standard errors, dependent variable: performance improvement, 96 observations.*

	ARIMA vs. ARIMASVR (hyp. 3,5)			**SVR vs. SVRARIMA (hyp. 4,6)**		
Variable	Coeff.	St. E.	P–value	Coeff.	St. E.	P–value
intercept	–0.13	2.960	4.60e–05 ***	0.32	0.828	0.70
sd	1.55	0.585	0.0098 ***	0.10	0.100	0.30
sd^2	–0.03	0.055	0.6200	0.00	0.007	0.94
sd^3	0.00	0.002	0.6318	0.00	0.000	0.83
mean	1.05	0.003	0.0005 ***	–0.05	0.10	0.07
$mean^2$	–0.03	0.011	0.0017 **	0.00	0.003	0.60
$mean^3$	0.00	0.000	0.0016 **	0.00	0.000	0.81
sd×mean	–0.03	0.006	5.48e – 06 ***	0.00	0.002	0.67
	$R^2 = 0.83$, adj. $R^2 = 0.81$ F–statistic: 58.64 on 7 and 88 DF			$R^2 = 0.11$, adj. $R^2 = 0.04$ F–statistic: 1.53 on 7 and 88 DF		
sd: average 9.3, standard dev. 8.1 mean: average 30, standard dev. 9.0						

Source: the author.

that the best model identified in this research is the optimal forecasting technique. In order to address the optimal model, other approaches should be tested, especially machine learning tools capable of capturing highly nonlinear behavior, e.g., gradient boosting methods or random forests, which are still relatively undiscovered by the EPF literature. This remains for further investigation.

A second limitation of this study lies in omitting general equilibrium effects. Changing transmission constraints has likely many consequences, e.g., higher forecasting precision that decreases the final price due to lower balance costs, which are not considered here and which might affect the forecasting precision. In addition, the supplied quantity of electricity affects the resulting price, hence production planning of an agent. In other words, if a large producer increases the amount of electricity supplied, the forecasted quantity should increase and the price on the market should decrease. However, effects of individual agents are considered in this research to be sufficiently small, which is plausible for small- to medium-size producers. One can argue for similar effects on the demand side in the case of large industrial firms. However, the problem on the demand side seems to be even less important since the fraction of electricity consumed is rather small due to households being the major consumer.

CONCLUSION

The aim of the paper is to investigate circumstances under which the hybrid ARIMA-SVR models yield superior performance over separate use of each method in the context of short-term electricity price forecasting. In conclusion, using combination of ARIMA and SVR leads to increased forecasting performance, which is more pronounced the higher the volatility and the higher the transmission congestion when the congestion is measured by first and second moments of price due to the structure of the Nord Pool. When the RMSE is of interest, hybrid ARIMA-SVR methods appear to be clearly superior to single ARIMA or SVR. Interestingly, when the MAE is of interest, a single SVR model delivers the best performance in many cases. Overall, the performance of methods employing SVR is very similar. This finding suggests that the additional flexibility of hybrid methods over SVR is beneficial for capturing price spikes at a slight cost of imprecise forecasting during steady periods. Using a single ARIMA under STL decomposition seems not suitable for forecasting due to its inability to model irregularities, which are frequent in deregulated electricity markets.

The forecasting performance is lower for areas with higher transmission congestion. One part of the total costs of transmission constraints can thus also be seen as a cost of a higher forecasting error. By a coarse calculation ignoring other effects, the average costs of congestion for the Nord Pool in 2013–2016 seem to be about €0.36/MWh.

The ARIMA-SVR models represent a suitable method for electricity price forecasting, nevertheless, other methods should also be considered in the search for feasible, reliable, and precise hourly electricity price forecasting methods. The analysis suggests that especially approaches capable of modeling highly nonlinear behavior could yield decent performance, e.g., gradient boosting methods or random forests.

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