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APPLICATION OF PERFORMANCE RATIOS IN PORTFOLIO OPTIMIZATION

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Abstract

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The cornerstone of modern portfolio theory was established by pioneer work of Harry Markowitz. Based on his mean-variance framework, Sharpe formulated his well-known Sharpe ratio aiming to measure the performance of mutual funds. The contemporary development in computer's computational power allowed to apply more complex performance ratios, which take into account also higher moments of return probability distribution. Although these ratios were proposed to help the investors to improve the results of portfolio optimization, we empirically demonstrated in our paper that this may not necessarily be true. On the historical dataset of DJIA components we empirically showed that both Sharpe ratio and MAD ratio outperformed Rachev ratio. However, for Rachev ratio we assumed only one level of parameters value. Different set-ups of parameters may provide different results and thus further analysis is certainly required.

Keywords: portfolio optimization, Sharpe ratio, mean absolute deviation ratio, Rachev ratio, efficient market hypothesis, time series modelling, GARCH model, copula function

INTRODUCTION

The portfolio optimization represents a classical financial problem in which practitioners and theorists search for the answer to the question how to choose the best portfolio composition. Although the cornerstone of modern portfolio theory was set in 1952 by Harry Markowitz, there is still broad discussion in scientific literature about portfolio optimization topic and there are published many empirical studies such as e.g. Širůček and Křen (2015).

The process of selecting the portfolio may be divided into two stages. In the first stage, the historical performance of available assets is studied and the future performance is predicted based on the available data. In the second stage, the choice of optimal portfolio is made based on the predicted future development. As both stages represent a challenging task we discuss them in a greater depth.

Contemporary state of the art of financial time series modelling is connected to the Efficient Market Hypothesis (henceforth EMH) according to which "prices fully reflect all available information" and hence future evolutions are unforecastable, see e.g. Samuelson (1965). In simple terms, EMH states that by predicting the future development we are not able to achieve the profits superior to the profits of the market index when these are adjusted for the risk and transactions costs are deducted. Under this hypothesis, the investors can achieve higher profits only by undertaking higher risks – under EMH the return and the risk of the investments (stocks) can be modelled. However, when modelling the asset returns, even in efficient markets there exist empirical properties such as fat-tailed distribution, volatility clustering, gain/loss asymmetry, leverage effect and dependence in tails, which make modelling of return time series and subsequent estimation of the future performance a challenging task, see e.g. Cont (2001).

On the other hand, there are works such as Lo and MacKinlay (2011) and Lo *et al.* (2000) providing evidences that markets are not efficient. In these works, however, the strategies (or technical trading rules) are demonstrated to provide the extra performance in short term only and then the extra performance vanishes. The answer to this discrepancy can be adaptive markets hypothesis introduced by Lo (2004). Thus, in our paper we assume the returns to follow autoregressive model with stochastic volatility modelled by GARCH model. The autoregressive part of the model allows the predictability of the future returns to a certain extent, but this does not necessarily mean the violation of EMH as the higher expected returns can be compensated by the increased risk (represented by the volatility).

However, even when we form our beliefs about the future evolution of returns, the task to choose optimal portfolio is not easy. The cornerstone in this field was a pioneer work of Markowitz (1952). In his work, Markowitz considered expected return a desirable thing and variance of return an undesirable thing. His choice of variance was purposely because of its computational simplicity. However, nowadays we can argue that the choice of semi-variance would better reflect the risk of the portfolio. Nevertheless, based on Markowitz mean-variance framework, Sharpe (1966) introduced a well-known Sharpe ratio, by which different portfolios can be ranked based on the ratio of expected excess return and its standard deviation. The drawback of the Sharpe ratio as well as Markowitz mean-variance framework is that they do not consider higher moments of return probability distributions. In order to overcome this imperfection there were introduced many other performance ratios which differ in applied measures or risk and reward. The examples are e.g. Gini ratio (Shalit and Yitzhaki, 1984), mean absolute deviation ratio (Konno and Yamazaki, 1991), mini-max ratio (Young, 1998), Rachev ratio (Biglova *et al*., 2004) and others, for the summary see e.g. Farinelli *et al*. (2008).

However, as was stated above, the process of selecting the portfolio composition consists of two stages. While the focus in the present literature is given mostly to the development of new models and more complex ratios applicable in the second stage of decision-making process, little attention is given to the first stage and even to the question to which extent we are able to predict the complex measures of risk such as Conditional Value at Risk utilized in Rachev ratio. The same question arises, if we base our decision-making on the historical returns. These returns might not necessarily provide any useful information about the risk in the future. On the other hand, the simple risk measures (such as standard deviation of returns) can prove their robustness and the ratios based on them can provide the same or better results than the more complex ratios.

The aim of the paper is to backtest portfolio optimization assuming selected ratios and compare the obtained results. In order to simulate the future performance (i.e. returns) two approaches are considered: historical data and estimation/ simulation of GARCH-copula model. The dataset which is applied in the performed analysis consist of stocks incorporated in Dow Jones Industrial Average and covers the period from April 8, 2004 until December 31, 2014.

The paper is structured as follows. In the next section we describe methods and data applied in the paper. To be concrete, we define the GARCH-copula model, portfolio optimization problem with applied performance ratios and the methodology of the paper with applied dataset. In the next two sections the obtained results are presented and discussed. Firstly, we present the results of particular performance ratios without deducing transaction costs and then we study the influence of transaction costs on the best portfolio optimization strategy. The last section provides the conclusion.

MATERIALS AND METHODS

Modelling of Financial Asset Returns

Evolution of financial asset returns over time is specific in the following ways, for further details see e.g. Cont (2001). Empirical volatility of returns is not constant over time, but is rather clustered. Thus, for the same asset, the periods of high volatility (high gains/losses) can be seen as well as the periods in which volatility is low (the gains/losses are low). This issue can be tackled by the volatility modelling, in the paper we particularly apply GARCH model introduced by Bollerslev (1986). Even after the correction of returns for volatility clustering, the residual time series still exhibit heavy tails. The conditional distribution, however, is less heavy-tailed than unconditional distribution. In our paper we utilize joint Student distribution for residuals. Due to the estimation and simulation requirements, this joint distribution is decomposed into Student marginal distributions and Student copula function in line with Sklar's theorem (Sklar, 1959). We address the obtained model as GARCH-copula model, which was already successfully applied in risk management, see e.g. Huang *et al*. (2009), Wang et al. (2010) or Klepáč and Hampel (2015).

Assume that we want to model the future returns of n assets. For each return time series we assume AR(1)-GARCH(1,1) process, i.e. i-th asset returns can be modelled as follows,

$$r_{i,t} = \mu_i + \gamma_i \cdot r_{i,t-1} + \sigma_{i,t} \cdot \varepsilon_{i,t}, \tag{1}$$

where μ_i and γ_i are the parameters of conditional mean model, $\sigma_{i,t}$ is standard deviation (volatility) modelled by the GARCH model and $\varepsilon_{i,t}$ is a random number from Student probability distribution (henceforth filtered residual). The Student distribution is applied for its ability to model the fat tails (higher kurtosis) of probability distribution, which are usually present in return time series. The volatility is modelled by means of GARCH model (Bollerslev, 1986), an extension of ARCH model (Engle, 1982). The applied model takes the following form,

$$\sigma_{i,t}^2 = \kappa + \alpha_i \cdot \sigma_{i,t-1}^2 + \beta_i \cdot \sigma_{i,t-1}^2 \cdot \varepsilon_{i,t-1}^2, \tag{2}$$

where κ, α_i and β_i are the parameters needed to be estimated. The positive variance is assured if all the parameters are equal or greater than zero. Model is stationary if $\alpha_i + \beta_i < 1$.

In order to preserve the mutual dependence among the asset returns the filtered residuals are joined together applying copula function modelling. Copula functions are projections of the dependency among particular distribution functions into [0, 1],

$$C: [0, 1]^n \to [0, 1] \text{ on } R^n, n \in \{2, 3, \ldots\}. \quad (3)$$

Basic reference for the theory of copula functions is Nelsen (2006), while Rank (2007) and Cherubini *et al.* (2004, 2011) target mainly on the application issues in finance. Actually, any copula function can be regarded as a multidimensional distribution function with marginals in the form of standardized uniform distributions. Following the Sklar's theorem, see Sklar (1959, 1973), any joint distribution function, in our case the joint distribution function of filtered residuals $F_{\varepsilon_{1,t}, \ldots, \varepsilon_{n,t}}(x_1, \ldots, x_n)$ can be decomposed into marginal distributions and selected copula function,

$$F_{\varepsilon_{1,t}, \ldots, \varepsilon_{n,t}}(x_1, \ldots, x_n) = C(F_{\varepsilon_{1,t}}(x_1), \ldots, F_{\varepsilon_{n,t}}(x_n)) = C(u_1, \ldots, u_n). \quad (4)$$

The formulation above should be understood such that the copula function *C* specifies the dependency, nothing less, nothing more. With some simplification, we can distinguish copulas in the form of elliptical distributions and copulas from the Archimedean family. While the first group is defined by means of related joint distribution function and the copulas are generally *n*-dimensional, the Archimedean copulas are defined by means of generator function and these copulas are usually defined as 2-dimensional functions. The extension to *n*-dimensional case is not straightforward and these copulas must be nested into each other or grouped together (hierarchical Archimedean copulas or vine copulas). The exemplary applications of these approaches are given e.g. by Kubát and Górecki (2015) and Klepáč and Hampel (2015). As both approaches would increase time complexity of the estimation/simulation procedure and thus the time demands for the backtesting computations would become enormous, we apply Student copula function (elliptical copula). Note that by accompanying Student copula function with Student marginals we obtain the joint-Student distribution.

Portfolio Optimization Problem

In this paper we assume the investor maximizing selected performance ratio, i.e. solving following portfolio optimization problem,

$$\mathbf{w} = \begin{cases} \arg\max_{\mathbf{x}} PR(\mathbf{R} \times \mathbf{x}) \\ \sum_{i=1}^{n} x_i = 1 \\ x_i \geq 0, i = 1, \ldots, n \\ x_i \leq 0.25, i = 1, \ldots, n \end{cases} \quad (5)$$

in which **x** is the vector of weights (portfolio composition) and **R** represents the matrix of random realizations of returns. We can distinguish two approaches to the optimization problem:

- in the first approach we utilize historically observed returns over selected period (perform ex-post optimization and assume that the future performance of each stock will not change),
- in the second approach we simulate future returns by means of selected parametrical model such as above described GARCH-copula model.

In both approaches, rows in the matrix **R** represent realizations with equal probability and columns represent particular assets the investor can include in the portfolio. Furthermore, the constraints of the optimization problem bound the weight of each asset between 0% (short selling is not allowed) and 25% (the portfolio is composed of at least four assets).

The most known performance ratio is Sharpe ratio which is closely related to Markowitz mean-variance framework as it focuses only on the first two moments of probability distribution. However, as it is known, the empirical distribution of financial asset returns is characteristic by heavy-tails and skewness. Thus, many researchers proposed their own ratios, which take into account also the kurtosis and skewness of probability distribution. Among others, see for instance Gini ratio (Shalit and Yitzhaki, 1984), mean absolute deviation ratio (Konno and Yamazaki, 1991), mini-max ratio (Young, 1998), Rachev ratio (Biglova *et al.*, 2004) and others. For the summary see Farinelli *et al.* (2008).

Sharpe (1966) continued in the framework established by Markowitz and proposed the well-known Sharpe ratio (Sharpe index, the Sharpe measure or the reward-to-variability ratio) which he first defined as the ratio between the excess expected return (i.e. the expected return minus risk-free rate; also known as risk premium) and its volatility,

$$SR(\tilde{r}) = \frac{E(\tilde{r} - r_{RF})}{\sigma_{\tilde{r} - r_{RF}}} = \frac{E(\tilde{r}) - r_{RF}}{\sigma_{\tilde{r}}}, \quad (6)$$

where $\tilde{r}$ is observed (or predicted) distribution of returns or equiprobable realizations of this distribution and r_{RF} is risk-free rate. The original ratio was revised by Sharpe (1994) substituting the risk-free rate by an applicable benchmark $\tilde{r}_B$, which can change in time,

$$SR(\tilde{r}) = \frac{E(\tilde{r} - \tilde{r}_B)}{\sigma_{\tilde{r} - \tilde{r}_B}}. \quad (7)$$

Further in this paper we assume the original version of Sharpe ratio (6), which in fact is a special case of the revised version (7) in which $\tilde{r}_B = r_{RF}$. The Sharpe ratio defines the profile of an investor who prefers titles with higher expected excess returns for unity of volatility (standard deviation). When comparing two assets versus a common benchmark (in our case risk-free rate), the one with higher Sharpe ratio provides better return for the same risk (or, equivalently, the same return for lower risk).

Mean absolute deviation ratio (henceforth MAD) considers expected return as a desirable thing and mean absolute deviation from the expected return as an undesirable thing,

$$MAD(\tilde{r}) = \frac{E(\tilde{r}) - r_{RF}}{E\left[\left|\tilde{r} - E(\tilde{r})\right|\right]}. \quad (8)$$

The ratio still does not take the higher moments of probability distribution into account. Thus, it can be considered as the different version of Sharpe ratio.

The Rachev ratio (Biglova *et al.*, 2004) is defined as follows:

$$RR_{\alpha,\beta}(\tilde{r}) = \frac{CVaR_\alpha(-\tilde{r} + r_B)}{CVaR_\beta(\tilde{r} - \tilde{r}_B)}, \quad (9)$$

where $\tilde{r}_B$ is a benchmark return that we assume to be equal to r_{RF},

$CVaR_\beta(X) = \frac{1}{\beta}\int_0^\beta VaR_u(X)du$ is the Conditional Value at Risk of random variable X and $VaR_u(X)$ is the Value at Risk of the random variable X. The Conditional Value at Risk $CVaR_\beta(X)$ is a coherent risk measure (see Artzner *et al.* 1999 and Rockafellar and Uryasev, 2002) and it is the opposite of the mean of the returns below the percentile of its distribution. Thus, Rachev ratio allows to optimize the trade-off between maximum gains and losses.

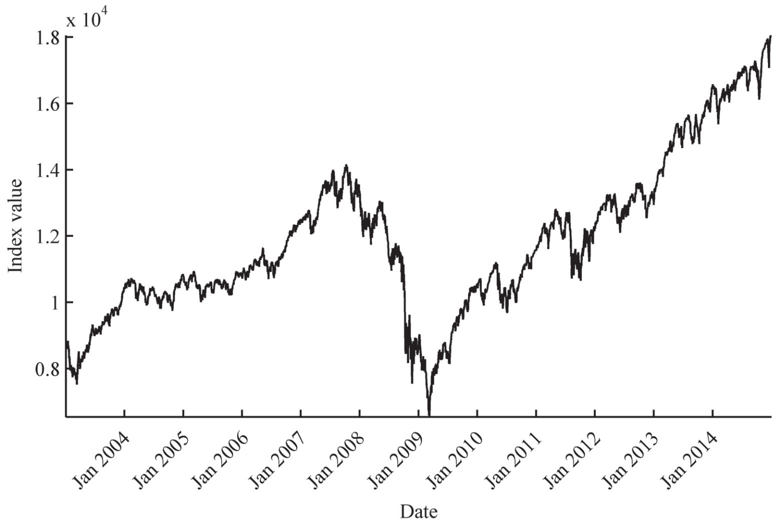
Dataset

The utilized dataset consists solely of the stocks incorporated in one of the American stock market indices – Dow Jones Industrial Average (henceforth DJIA). In order to avoid the survivorship bias, we assumed all the components of the index as of April 8, 2004. Historical data of the stocks included in the dataset were obtained from Yahoo Finance website over the period from January 2, 2003 until December 31, 2014 (3,020 daily observations for each stock). However, we estimated the parameters from 250 observations, which must have been left for initial parameters estimation. Thus, the backtesting were performed in the period from April 8, 2004 (the date from which the composition of the DJIA index was taken) until December 31, 2014 (2,700 daily observations).

The evolution of DJIA index in the analysed period (2003–2014) is depicted in Fig. 1. The index took the value of 8,607.52 on January 2, 2003 and 17,983.07 on December 31, 2014. The average annual return (to be more specific the average return of 250 trading days) in the analysed period was 6.29% whereas the maximum drawdown over the analysed period was 53.78% (in years 2008–2009).

Methodology

In the paper we verify the proposed portfolio optimization procedure and compare the results obtained by means of application of selected performance ratios. We proceeded as follows. The portfolio optimization is performed on rolling window basis over analysed period, i.e. for each day we consider preceding 250 observations and we optimize portfolio composition by means of two approaches:



1: *Evolution of DJIA index in the analysed period*
Source: The author's own elaboration of data obtained from http://finance.yahoo.com

- assuming historical performance, i.e. the matrix **R** is composed of preceding 250 observations of assets' returns,
- applying GARCH-copula model, see equations (1), (2), (4), by which we simulate 25,000 random one-day-ahead future returns, i.e. the matrix **R** is composed of 25,000 simulated one-day ahead returns.

For both approaches we solve each day the portfolio optimization problem (5) taking into account historical returns or the randomly simulated returns and maximizing particular performance ratio. The Applied risk-free rate is 0% in all performance ratios and parameters α, β in Rachev ratio are 5%. The value of 5% was taken in accordance with the previously published researches. The optimization of the non-linear problem (5) was performed in Matlab utilizing function *fmincon* and assuming 20 random starting points (in order to find global instead of local maximum).

As the portfolio optimization problem is applied on the moving window basis, i.e. for each day the portfolio composition is calculated from the previous data, we can compute the ex-post wealth path W_t,

$$W_{t+1} = W_t \cdot \sum_{i=1}^{n} (r_{i,t} \cdot w_{i,t} - c_t), \tag{10}$$

where $r_{i,t}$ are ex-post observed returns, c_t are transaction costs and $w_{i,t}$ is the weight of the i-th asset at time t (portfolio composition) – these weights were obtained by means of maximizing selected performance ratio, see problem (5).

In our analysis we set the initial wealth (the wealth at the beginning of analysed period) to 1. By repeatedly applying formula (10) we compute the wealth path evolution over the analysed period and the final wealth at the end of analysed period. In our analyses we neglected the transaction costs (c_t). The influence of transaction costs is further studied only for the best portfolio optimization strategy.

The computations were performed in Matlab. Some applied algorithms (general backtesting framework) are described in Kresta (2015a, 2015b). Algorithms for calculation and optimization of MAD ratio and Rachev ratio were coded and are freely available upon an e-mail request to the author.

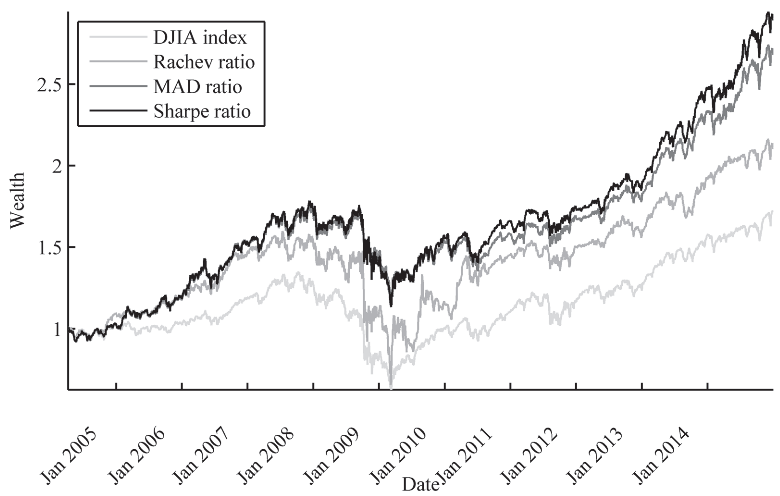
RESULTS

In this section we present the obtained results. Firstly, we show the results of portfolio optimization assuming historical returns, then the results when applying GARCH-copula model and finally we study the influence of the transaction costs on the results.

Historical Performance

The empirical results of the portfolio optimization backtesting are depicted in Fig. 2. In the figure the wealth paths obtained by maximizing particular performance ratios are compared to each other and to the investment into the DJIA index. As can be seen from the figure the active portfolio management following proposed portfolio optimization procedure outperforms the passive investment into DJIA index. The highest final wealth is obtained by means of maximizing Sharpe ratio, the second is the MAD ratio and the worst performance is obtained by means of Rachev ratio.

In Tab. I we summarize the obtained final wealth and maximum drawdown of different strategies (applying different performance ratios). From the results we can confirm that the Rachev ratio provides the worst performance – the final wealth is the lowest while the maximum drawdown is the highest. Both Sharpe and MAD ratios provide similar performance. Although Sharpe ratio provides higher value of the final wealth (annual

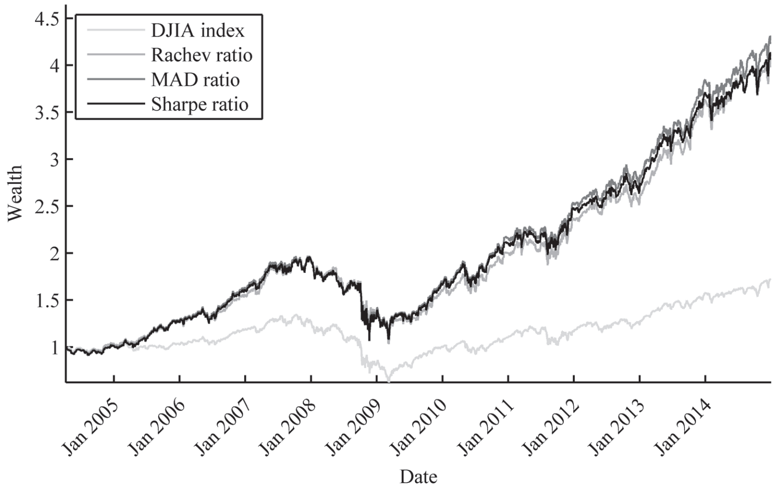


2: *Wealth paths obtained by means of maximizing particular performance ratios*
Source: The author's own calculations

I: *Final wealth and maximum drawdown of particular wealth paths*

	Sharpe ratio	MAD ratio	Rachev ratio	DJIA
Final wealth	2.891	2.680	2.103	1.716
Annual return	10.32%	9.55%	7.12%	5.12%
Maximum drawdown	36.52%	34.49%	58.97%	53.78%

Source: own elaboration



3: *Wealth paths obtained by means of maximizing particular performance ratios*
Source: The author's own calculations

II: *Final wealth and maximum drawdown of particular wealth paths*

	Sharpe ratio	MAD ratio	Rachev ratio	DJIA
Final wealth	4.059	4.228	3.980	1.716
Annual return	13.85%	14.28%	13.64%	5.12%
Maximum drawdown	45.63%	45.60%	47.67%	53.78%

Source: The author's own calculations

return respectively), also the maximum drawdown is higher compared to the MAD ratio.

The transaction costs are not subtracted. The reason is twofold. Firstly, they differ significantly – although they would represent high fraction of the gains for small private investors, for large institutional investors they would be of smaller values. Secondly, also the considered investments into DJIA index is connected with trading costs which are caused by the changes in stock prices (and thus also changes in relative weights).

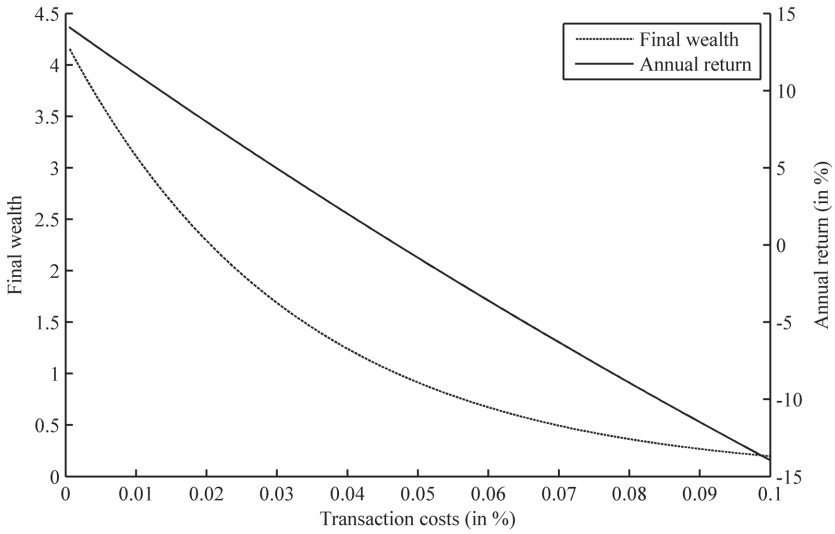
GARCH-copula Prediction

The empirical results of the portfolio optimization backtesting are depicted in Fig. 3. In the figure the wealth paths obtained by maximizing particular performance ratios are compared to each other and to the investment into the DJIA index. As can be seen from the figure the active portfolio management following proposed portfolio optimization procedure outperforms the passive investment into DJIA index. Moreover, we can see that there are not significant differences in wealth paths obtained considering different performance ratios.

In Tab. II we summarize the obtained values of final wealth and maximum drawdown of different strategies (applying different performance ratios). See that, while investing into DJIA index the investor would multiply his wealth by 1.7 (annual return of 5%), following proposed active strategies the wealth would be quadrupled (annual return of 14%). However, the transaction costs are not subtracted.

The next factor the investor is concerned about is the risk of the strategy. In Tab. II we also recorded the maximum drawdown of particular strategies in analysed period. It shows up that not only the proposed active strategies outperformed the index in terms of final wealth but also the maximum drawdown decreased. While the index lost almost 54% of its value during the financial crisis in 2008–2009, the maximum drawdown of the active strategies was around 46%.

When comparing particular performance ratios, we can see two things. The differences are small and the results obtained by means of Rachev ratio



4: *The values of final wealth and annual return in dependence on transaction costs*
Source: The author's own calculations

maximization are worse than applying conservative performance ratios. However, different values of parameters may produce better results and thus the further research is needed.

Influence of the Transaction Costs on Final Wealth

It is also important to study the influence of transaction costs on the final wealth. In previous figures we haven't assumed the transaction costs, i.e. $c = 0$ in (6). In Fig. 4 we show the results of the MAD strategy when applying GARCH-copula approach (the best strategy) assuming different transaction costs. Transaction costs are assumed both for buying and selling and are applied only for changes in portfolio composition (e.g. if the weight of one asset in portfolio decreases from 25% to 15% the transaction costs are calculated only for the difference, i.e. for 10%).

We are interested in two thresholds: i) the final wealth (annual return) equals to the value of DJIA investment and ii) final wealth equals to 1 (i.e. annual return equals to zero). As can be seen from the figure, the active portfolio optimization strategy is profitable for transaction costs lower than 0.047% and outperforms the DJIA investment strategy for transaction costs lower than 0.03%. It is obvious that such a small transaction costs will not apply for a private investor. On the other hand the proposed active portfolio optimization strategy can be interesting for institutional investors.

DISCUSSION

In Fig. 2 and Fig. 3 we have shown the wealth paths of proposed active portfolio optimization strategies and compared them with the passive investment into DJIA index. As was shown, the active portfolio strategies outperformed the passive strategy both in terms of final wealth (annual return respectively) and maximum drawdown. Although it may seem that our findings are not in accordance with efficient market hypothesis, the transaction costs must not be overlooked. As it turns out the transaction costs must have been lower than 0.03% for MAD ratio strategy in order to outperform the passive investment strategy, see Fig. 4. Such a small transaction costs will not apply for a private investor. Considering institutional investors it is questionable whether their transaction costs are lower. The observed high influence of transaction costs on the value of final wealth is due to the frequent changes in portfolio composition – we observed that the portfolio composition changes completely each few days (3–5 days). Thus, we can conclude that the obtained results are in accordance with efficient market hypothesis as it is not possible to obtain the profits superior to the profits of the market index when transaction costs are subtracted (considering real transaction costs higher than 0.03%).

Moreover we compared application of different performance ratios in portfolio optimization. As it is clear from the comparison of wealth paths in Fig. 2 and Fig. 3 there are not significant differences depending on the fact which performance ratio is applied. Although since Sharpe (1966) there were defined many new performance ratios, each performance ratio being more and more complex, in our empirical analysis we found out that they must not necessarily provide better results. However, the topic of performance ratios application is still in current interest of researchers, see e.g. Giacometti *et al.* (2015). The tendency is to generalize the formulas for performance ratio incorporating more parameters in them, e.g. parameters α and β in Rachev ratio. This generalization brings the need to set the values to these parameters. In our paper we applied the value of 5% both for parameter α and β

according to the original proposition of the Rachev ratio (Biglova *et al.*, 2004). However, different values may provide different results. The future research area can be the analysis of different values of these parameters and their influence on wealth path evolution. However when analysing ex-post data one have to be careful not to succumb the data snooping bias.

CONCLUSION

The cornerstone of modern portfolio theory was set by Markowitz in 1952 and the portfolio optimization problem is in the constant focus of both academics and practitioners. The development in computer's computational power allowed to apply more complex performance ratios, which take into account also higher moments of return probability distribution.

Although these ratios were proposed to improve the portfolio allocation, we empirically demonstrated in our paper that they may not necessarily provide an advantage over the simpler ones. On the historical dataset of DJIA components we empirically showed that both Sharpe ratio and MAD ratio outperformed Rachev ratio. The explanation that comes to our mind is that the simpler ratios are more robust and less sensitive to the inaccurate predictions of future performance. However, in our empirical analysis we assumed only one level of parameters' values in Rachev ratio. Different set-ups of parameters may provide different results and thus there is a need for further analysis.

In the practical applications not only the choice of particular performance ratio is important but also the way the future development is predicted. In the paper we backtested different approaches to portfolio optimization problem. We assumed selected performance ratios and two approaches to the optimization: historical data and estimation/simulation of GARCH-copula model. We found out that the better results are obtained when applying latter approach – the investor increases the wealth more and also decreases the drawdown. Thus, we can conclude that it is better to estimate the parametric model and simulate the future returns than to optimize the ratios only on historical returns and believe that the performance of individual assets will repeat. We can conclude that the applied GARCH-copula model provides a better estimate of future development.

As the empirical results suggested by following proposed active portfolio strategy the investor increases his/her wealth and decreases the drawdown (if the transaction costs are omitted), which seems to contradict the efficient market hypothesis, we focused also on the influence of the transaction costs on the final wealth. We found out that the best of proposed active strategies outperforms the investment in Dow Jones Industrial Average only if transaction costs are lower than 0.03%. Thus, if the investor's transaction costs are higher than 0.03%, then the passive investment strategy should be preferred.

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REFERENCES

ARTZNER, P., DELBAEN, F., EBER, J. M. et al. 1999. Coherent measures of risk. *Mathematical Finance*, 9(3): 203–228.

BIGLOVA, A., ORTOBELLI, S., RACHEV, S. T. et al. 2004. Different approaches to risk estimation in portfolio theory. *The Journal of Portfolio Management*, 31(1): 103–112.

BOLLERSLEV, T. 1986. Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31(3): 307–327.

CONT, R. 2001. Empirical properties of asset returns: stylized facts and statistical issues. *Quantitative Finance*, 1(2): 223–236.

ENGLE, R. F. 1982. Auto-regressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, 50(4): 987–1007.

FARINELLI, S., FERREIRA, M., ROSSELLO, D. et al. 2008. Beyond Sharpe ratio: Optimal asset allocation using different performance ratios. *Journal of Banking & Finance*, 32(10): 2057–2063.

GIACOMETTI, R., ORTOBELLI, S. and TICHÝ, T. 2015. Portfolio Selection with Uncertainty Measures Consistent with Additive Shifts. *Prague Economic Papers*, 24(1): 3–16.

HUANG, J. J., LEE, K. J., LIANG, H. et al. 2009. Estimating value at risk of portfolio by conditional copula-GARCH method. *Insurance: Mathematics and Economics*, 45(3): 315–324.

CHERUBINI, U., LUCIANO, E. and VECCHIATO, W. 2004. *Copula Methods in Finance*. Chichester: Wiley.

CHERUBINI, U., MULINACCI, S., GOBBI, F. et al. 2011. *Dynamic Copula Methods in Finance*. Chichester: Wiley.

KLEPÁČ, V. and HAMPEL, D. 2015. Assessing Efficiency of D-Vine Copula ARMA-GARCH Method in Value at Risk Forecasting: Evidence from PSE Listed Companies. *Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis*, 63(4): 1287–1295.

KONNO, H. and YAMAZAKI, H. 1991. Mean-absolute deviation portfolio optimization model and its applications to Tokyo stock market. *Management Science*, 37(5): 519–531.

KRESTA, A. 2015a. *Financial Engineering in Matlab: Selected Approaches and Algorithms*. Ostrava: VŠB-TU Ostrava.

KRESTA, A. 2015b. Application of GARCH-Copula Model in Portfolio Optimization. *Financial Assets and Investing*, 6(2): 7–20.

KUBÁT, M. and GÓRECKI, J. 2015. An experimental comparison of Value at Risk estimates based on elliptical and hierarchical Archimedean copulas. In: *33rd International Conference Mathematical Methods in Economics: Conference Proceedings*. Cheb, 9–11 September. Plzeň: University of West Bohemia, 419–424.

LO, A. W. 2004. The Adaptive Markets Hypothesis. *The Journal of Portfolio Management*, 30(5), 15–29.

LO, A. W. and MACKINLAY, A. C. 2011. *A Non-Random Walk Down Wall Street*. New Jersey: Princeton University Press.

LO, A. W., MAMAYSKY, H. and WANG, J. 2000. Foundations of technical analysis: Computational algorithms, statistical inference, and empirical implementation. *The Journal of Finance*, 55(4): 1705–1770.

MARKOWITZ, H. M. 1952. Portfolio selection. *Journal of Finance*, 7(1): 77–91.

NELSEN, R. B. 1997. Dependence and order in families of Archimedean copulas. *Journal of Multivariate Analysis*, 60(1): 111–122.

RANK, J. 2006. *Copulas: From Theory to Application in Finance*. London: Risk Books.

ROCKAFELLAR, R. T. and URYASEV, S. 2002. Conditional value-at-risk for general loss distributions. *Journal of Banking & Finance*, 26(7): 1443–1471.

SAMUELSON, P. A. 1965. Proof that properly anticipated prices fluctuate randomly. *Industrial Management Review*, 6(2): 41–49.

SHALIT, H. and YITZHAKI, S. 1984. Mean-Gini, Portfolio Theory, and the Pricing of Risky Assets. *The Journal of Finance*, 39(5): 1449–1468.

SHARPE, W. F. 1966. Mutual Fund Performance. *The Journal of Business*, 39(1): 119–138.

SHARPE, W. F. 1994. The Sharpe Ratio. *The Journal of Portfolio Management*, 21(1): 49–58.

SKLAR, A. 1959. Fonctions de repartition à n dimensions et leurs marges. *Publications de l'Institut de statistique de l'Université de Paris*, 8: 229–231.

SKLAR, A. 1973. Random variables, joint distribution functions, and copulas. *Kybernetika*, 9(6): 449–460.

ŠIRŮČEK, M. and KŘEN, L. 2015. Application of Markowitz Portfolio Theory by Building Optimal Portfolio on the US Stock Market. *Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis*, 63(4): 1375–1386.

WANG, Z. R., CHEN, X. H., JIN, Y. B. et al. 2010. Estimating risk of foreign exchange portfolio: Using VaR and CVaR based on GARCH–EVT-Copula model. *Physica A: Statistical Mechanics and its Applications*, 389(21): 4918–4928.

YOUNG, M. R. 1998. A minimax portfolio selection rule with linear programming solution. *Management Science*, 44(5): 673–683.

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