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ASSESSING EFFICIENCY OF D-VINE COPULA ARMA-GARCH METHOD IN VALUE AT RISK FORECASTING: EVIDENCE FROM PSE LISTED COMPANIES

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Abstract

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The article points out the possibilities of using static D-Vine copula ARMA-GARCH model for estimation of 1 day ahead market Value at Risk. For the illustration we use data of the four companies listed on Prague Stock Exchange in range from 2010 to 2014. Vine copula approach allows us to construct high-dimensional copula from both elliptical and Archimedean bivariate copulas, i.e. multivariate probability distribution, created from process innovations. Due to a deeper shortage of existing domestic results or comparison studies with advanced volatility governed VaR forecasts we backtested D-Vine copula ARMA-GARCH model against the VaR rolling out of sample forecast from October 2012 to April 2014 of chosen benchmark models, e.g. multivariate VAR-GO-GARCH, VAR-DCC-GARCH and univariate ARMA-GARCH type models. Common backtesting via Kupiec and Christoffersen procedures offer generalization that technological superiority of model supports accuracy only in case of an univariate modeling – working with non-basic GARCH models and innovations with leptokurtic distributions. Multivariate VAR governed type models and static Copula Vines performed in stated backtesting comparison worse than selected univariate ARMA-GARCH, i.e. it have overestimated the level of actual market risk, probably due to hardly tractable time-varying dependence structure.

Keywords: D-Vine copula, Christoffersen duration test, Kupiec test, Value at Risk, VAR-DCC-GARCH, ARMA-GARCH-GJR

INTRODUCTION

The increased level of financial or market risks and uncertainties is leading towards continuous development of more and more sophisticated methods serving for more accurate risk measurement and its management. This process is in motion via financial institutions, regulators and academic public. One of the most known approaches created for this purpose is the widely used metric Value at Risk (VaR) introduced in 1993, which measures the maximum possible loss on the given confidence level during the specific time period. Ergo while we forecast 1 day market VaR determined loss on confidence level 99% there is only 1% probability that loss overpass this forecast in the next 1 day. Financial institutions are probably primary users of the VaR metrics – due to its market risk exposures and obeying the Basel Accords, which explicitly recognizes the role of standard financial risk measures such as VaR. Financial institutions have to report in order to monitor their short term risk exposure and to compute the amount of economic capital subjected to regulatory control. The original applications in finance are heavily based on a lot of empirically rejected assumptions. Empirical researches have proved the non-normal distribution and other returns properties, e.g. autocorrelation of the residuals, heavy-tailed distributions of returns, time-varying volatility or correlation clustering in terms of multivariate models.

In this paper we concern many of these non-plausible theoretical assumptions, but in all of the modelling approaches we use *Generalized autoregressive conditional heteroskedasticity* (GARCH) volatility based techniques, when term volatility means the standard deviation of financial assets. This metric serves as an important input parameter of the risk measure used during valuation of many types of derivates and also in VaR calculations.

Proposal of well known (G)ARCH models of Engle (1982) and Bollerslev (1986) was used to account for volatility heterogenity in financial time series. After a while, numbers of multivariate extensions of GARCH(P,Q)[1] models have been introduced – mostly for dependence modelling of financial returns data, mostly for use in portfolio management or cross-country market analysis. In particular situations we can choose from the Constant Conditional Correlation GARCH model (CCC-GARCH) of Bollerslev (1990), the Dynamic Conditional Correlation GARCH (DCC-GARCH) of Tse & Tsui (2002) or Generalized Orthogonal GARCH model (GO-GARCH) proposed by Van der Weide (2002). For an summary about numerous number of existing (G)ARCH models see Bollerslev (2008). In the article we use to call them *standard models* and thus offer only limited explicit mathematical description, rather the reference to the previous application or definitions.

For better fit of models, and for treating of autocorrelation structure in raw data, we use commonly GARCH as conditional variance process and Autoregressive Moving Average (ARMA(M,N)) or Vector Autoregression (VAR(M)) as conditional mean. That means we work with ARMA(VAR)-GARCH, as in Dajcman & Kavkler (2011) or Angelidis, Benos & Deggianakis (2004).

Variations in GARCH models for VaR forecasting lead authors and quantitative risk management (QRM) experts to discussion when to use univariate or multivariate models. Univariate modelling is perhaps useful in cases when we analyze data without treating of the correlation structure between returns and in cases when we would not change weights in portfolio. On the other hand, in cases of the optimal portfolio selection with time-varying portfolio weights, it would be simpler to use multivariate models. Univariate models are thereby more suitable for estimation of VaR and multivariate models for portfolio selection. When dimension of the portfolio increases and computing time is an issue, the estimation of multivariate models becomes more complicated due to the large number of parameters. Bauwens, Laurent & Rombouts (2006) state that it is probably better to adopt univariate models. Our paper tests if it holds true also for our datasets or we should prefer VaR forecasts from multivariate models, for standard modelling approaches.

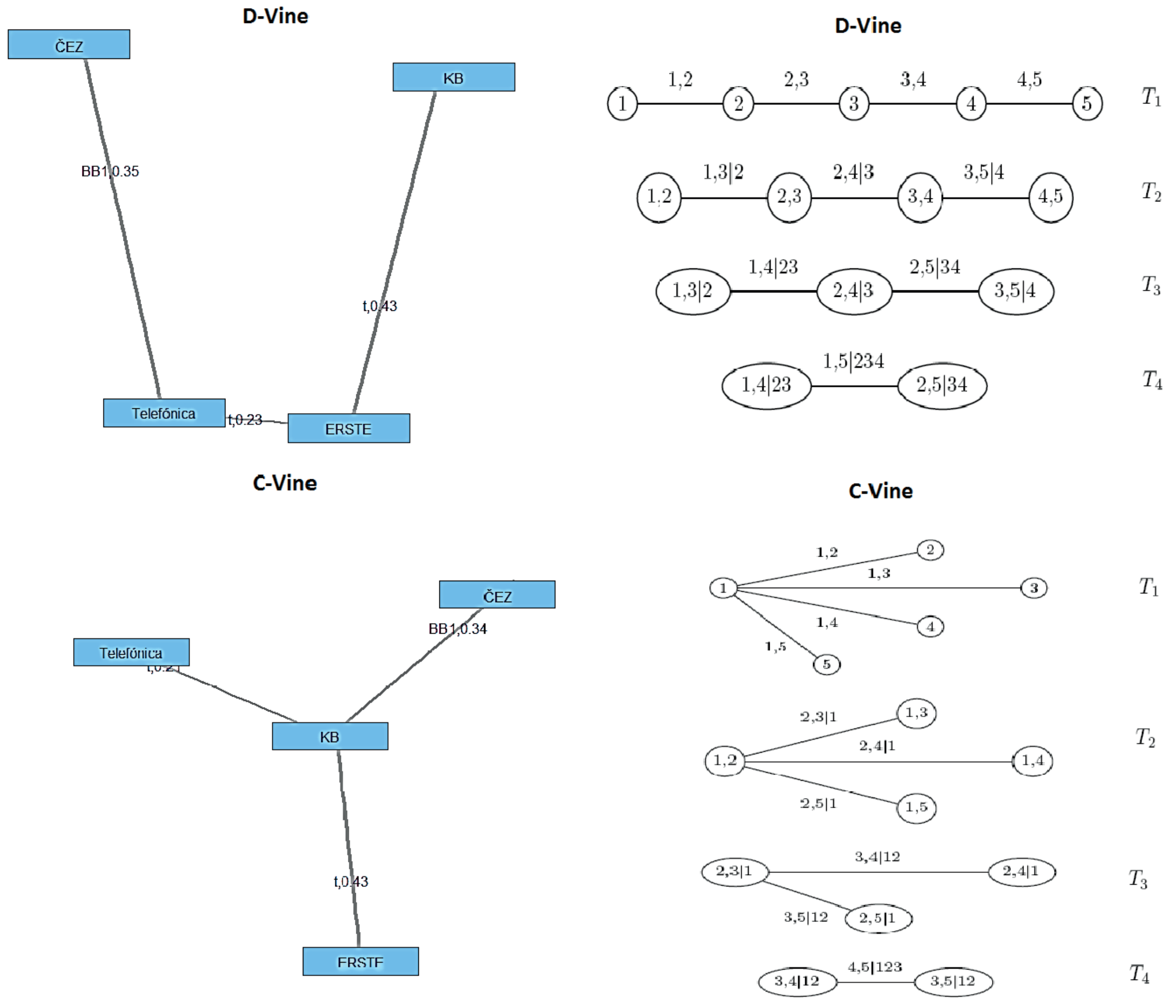
Another popular way of generalizing univariate GARCH models to multivariate dimension is connection with copulas for modeling of the dependence between inferred residuals as was previously used in Patton (2006) or in Min & Czado (2010). Copulas were firstly introduced in mathematical context by Sklar (1959) through his Sklar theorem. Any multivariate joint distribution can be written in terms of univariate marginal distribution functions and a copula describes the dependence structure between the variables. Continuous development of the copula theory supports us with solutions for elliptical and non-elliptical distributions as well, see Joe (1997) or Nelsen (1999) for mathematical reference about these copulas. For estimations we use maximum-likelihood estimation (MLE) – where copulas have one or two parameters. In this paper we are concerning use of full range of copula families from Brechmann & Schepsmeier (2013). Otherwise, the use of copulas is challenging in higher dimensions and therefor in QRM, where standard bivariate copulas suffer from inflexible dependence structures. For high dimensionality treatment many authors proposed pair-copula constructions (PCCs) class which structures partly shown in Fig. 1.

One of the specific types within this class are Vine copulas. Vines are a flexible class of n-dimensional dependency models when we use bivariate copulas as building blocks. Due to Aas *et al.* (2009) who described statistical inference techniques for the classes of canonical C-Vines and D-Vines, we can create multi-tier structure (according to dependence intensity between variables) between one central variable (i.e. market index) and underlying variables (companies in this index). D-Vines in contrast to its "sibling" offer another view: we propose modelling of the inner structure without selection of one explicit *dependence driver*.

Motivation and Contributions

The main aim of this paper is to test the prediction power in equally-weighted portfolio VaR one day ahead forecast, as important metric in financial industry, with D-Vine copula ARMA-GARCH model in comparison to selected univariate and multivariate models. Data consists of four companies listed on the Prague Stock Exchange: ČEZ, a. s., Telefónica Czech Republic, a. s., ERSTE Bank, a. s. and Komerční banka, a. s., from January 2010 to April 2014. On the basis of above stated we

1 P, Q from GARCH(P,Q) are signs for lag order of underlying variables in the models. It has similar meaning to lag orders M, N used in ARMA(M,N) process, see further text. For basic understanding see Engle (1982) and Bollerslev (1986).



1: *Examples of D-Vine tree (right upper panel) and C-Vine tree (right panel bellow) specification on five variables, on the left panels are samples from four-dimensional D-Vine (left upper panel) and C-Vine copula (left down panel). The both left panels show example of the first structure. Right panels according to approach Brechmann & Schepsmeier (2013).*

want to benchmark prediction abilities of D-Vines against other standard forecasting models: VAR-GO-GARCH, VAR-DCC-GARCH and univariate ARMA-GARCH model.

We approach and partly replicate research steps in Berg and Aas (2009) in the case of D-Vine copula ARMA-GARCH application. We also analyze D-Vine copula dependence structure of traded companies as in Allen *et al.* (2013). Similarly to the work of Angelidis, Benos & Degiannakis (2004) this paper uses the so-called conditional and unconditional coverage framework, where authors used for ARMA-GARCH analysis data of five European stock indices from 1987 to 2002. Dajcman & Kavkler (2011) used VAR-DCC-GARCH model to analyze comovements between Slovenian and other European stock markets on data from 1997 to 2010.

The results supply literature with at least two specific contributions: assessing of the best fitting standard and advanced D-Vine copula ARMA-GARCH models from many tested specifications and from data based on the chosen Czech listed companies. Secondary we tell if there exists significant difference in VaR forecasts between static univariate and multivariate approaches: both for static and dynamical dependence settings in case of VAR-GO-GARCH respectively VAR-DCC-GARCH.[2] Results then offer heavier concentration on the D-Vine copula model which in our best knowledge with regards to the mentioned standard volatility models and PSE listed companies, ain't been conducted before.

2 In the univariate case we use only one vector of portfolio returns for fitting and VaR forecasting. In the multivariate case is used one unique log returns vector for each company.

I: *Elementary statistics for log returns (from January 2010 to April 2014)*

Statistics	Telefónica	ČEZ	KB	ERSTE	E-W portfolio
Mean	0.000	0.000	0.000	2.75E-05	0.000
Stdev	0.013	0.014	0.018	0.026	0.012
Kurtosis	10.736	3.313	2.759	3.750	2.759
Skewnes	–1.190	–0.300	–0.147	–0.167	–0.204
Minimum	–0.102	–0.087	–0.108	–0.144	–0.059
Maximum	0.069	0.063	0.075	0.137	0.065

MATERIAL AND METHODS

We process data of time series of closing stock prices for four companies: ČEZ, a. s., Komerční banka, a. s., Telefónica O2 Czech Republic, a. s. and Erste Bank, a. s. which were obtained through Patria Online database, e.g. this contains data from Prague-XETRA system, denominated in CZK. In Tab. I, the basic statistics of logarithmic difference in prices (yields), because stock market data are usually non-stationary, for the time period from January 2010 to April 2014 are presented. We also present equally-weighted returns for portfolio consisting of four above mentioned companies.[3]

Representations of Volatility Models

Description about creating D-Vine copula ARMA-GARCH(1,1) model from Hofmann & Czado (2010) and Krause (2003) to establish univariate or multivarite GARCH models with mean process can be used for simple or multivariate processes either

$$X_{i,t} = \mu_{i,t} + \sqrt{h_{i,t} z_{i,t}} \,,$$

where
$z_{i,t}$...... draw from inverse cumulative distribution,
$h_{i,t}$...... conditional variance,
$\mu_{i,t}$...... conditional mean process,
$X_{i,t}$ .... a *i*-dimensional vector of return streams.

Basic GARCH (1,1) is then noted:

$$h_{i,t} = \omega_i + \alpha_i e_{i,t-1}^2 + \beta_i h_{i,t-1}, \qquad X_{i,0} = 0, \qquad h_{i,0} = 0,$$

where
$\omega_i > 0, \alpha_i \geq 0, \beta_i \geq 0,$
$e_{i,t}$ ...... residual element,
$h_{i,t}$...... denotes variance.

In case of D-Vine copulas the dependence structure among the residual components $e = (e_{1,t}, \ldots, e_{n,t})'$ for fixed t is assumed to be given by an n-dimensional D-Vine density from equation (11) in Czado (2010). Although, denoted GARCH(1, 1) form is just for illustration thus we could use more complex model structure, as in Bollerslev (2008). First equation above could be taken as abstract way how to connect both univariate and multivariate GARCH families. While $i = 1$ we speak about univariate stochastic proces as ARMA-GARCH. When we use $i > 1$ it means μ serves as VAR process with specific variance process component, finally as VAR-DCC-GARCH or VAR-GO-GARCH. See again Bollerslev (2002) and Van der Weide (2002) for description of the residual structure of variable e and h structure of standard volatility models used in this paper.

Selecting Best Fitting Models from in Sample Data

The final volatility model is chosen from standard GARCH, GJR, EGARCH, APARCH, iGARCH, which is tested in sample (seven hundred observation: from January 2010 to approx. October 2012)[4] on the basis of the minimal value of information criteria (Akaike (AIC) and Bayesian (BIC)) and t-statistic for different combination of lag orders (up to 5th order) together with ARMA process.

We test GARCH type governed innovations for different distributions: General error distribution (ged), Student-t, Hansen Skewed-t, Gaussian, Normal Inverse Gaussian (NIG). For assessing of the best fitting model from VAR-DCC-GARCH and VAR-GO-GARCH we use practicaly the same routine as in previous case – we aim to test many combinations of lag order (up to 5th lag) in VAR process with above stated GARCH variations. Although, we can use different correlation-wise lag orders in DCC(p,q) process we use DCC(1, 1) for reduction of results dimension. Procedures for estimation and forecasting of the multivariate volatility models are described more closely in Tse (2002) or Van der Weide (2002) for reference about used software (SW) see Ghalanos (2014a, 2014b).

To estimate D-Vine copula we should proceed in steps provided by Aas *et al.* (2009). At the first preparatory stage we filter (fit) raw data with ARMA model, standardize residuals by GARCH volatility – with the best fitting univariate models. Then we

3 The computations were performed with SW Matlab R2014b and R version 3.1.0 on basic data set of 1070 obervations for particular time series of log returns.
4 Recalibration of selected models was performed after 30 forecasting cycles, when we use model specification from initial selection. Best model should be selected daily after the moving window moves further for one step before actual forecast. That means 370 model fits and selections, which was not tractable for us with the full range of the used models.

transform the residuals into uniformly distributed data ~ [0, 1], we use algorithm tcdf in Matlab for this purpose. With data in this form we proceed acording to Aas *et al*. (2009) and conduct:

- Structure selection to assess the intensity and structure of dependence.
- Copula selection for the most appropriate fit of the tail dependence characteristics with Vuong-Clark test.
- Estimation of copula parameters with maximum-likelihood estimation (MLE), when we use copulas with one or two parameters.
- Model evaluation by Vuong test and sub-sequently by information criteria.
- Simulation from D-Vine copulas to get at least 10,000 times *n*-asset of uniformly distributed numbers.

Out of the Sample Forecasting and Backtesting of Estimates

After all of these previous steps we proceed to VaR rolling 1 day ahead forecasts, with rolling windows with length 700 days. For parametric model based on GARCH-type prediction we use the abstract distribution function G as proxy for tested types. We could casually use the notation like for one period forecast:

$$\mathrm{VaR}_{t+1} = \mu_{t+1} + \sigma_{t+1} \cdot G(q),$$

where

σ_{t+1}.... means the *p*-asset values in portfolio.

If the VaR for maximum losses on 95% confidence level as in our examples is required, the G(0.05) has to be inputted for calculation. Steps for forecasting of standard GARCH models are mostly described on reference of the used Ghalanos's SW packages in R (2014a, 2014b), which performs it automatically. For the VaR forecasting by D-Vine copula GARCH models we use more complex steps methodicaly described in Berg & Aas (2009):

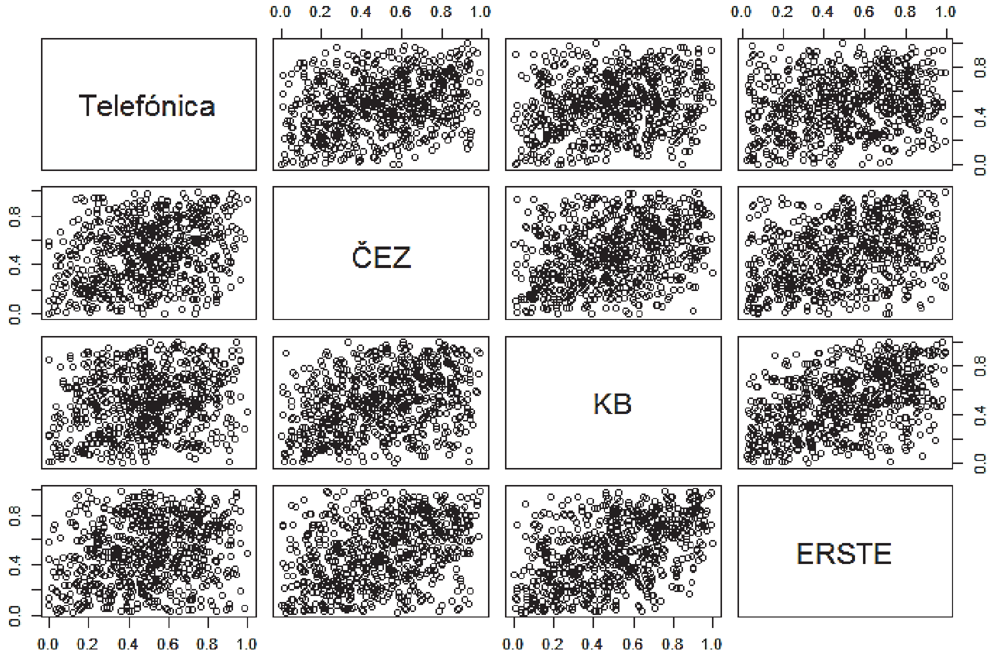
- Forecasts for full out of sample interval (370 forecasts) with conditional mean and volatility for each of companies: ARMA-GARCH best fitting models. ARMA conditional mean process should be used for better performance.
- Convert simulated [0, 1] uniformly distributed *U* (D-Vine copula-like) values into *G* distributions (returns-like), we use algorithm icdf in SW Matlab.
- Calculate the value of returns for each company in equity portfolio, with *n*-components, in $t + 1$ for all of the simulated uniformly distributed numbers. Calculate returns of the whole portfolio.
- Estimate the 95% VaR from simulated returns distribution in terms of the 5th quantile inference.

Because of the great importance of the quality of loss forecasts in financial industry there were developed backtesting procedures which validate use of the VaR estimators. Within the backtesting procedure see Christoffersen & Pelletier (2004) for conditional approach and Kupiec (1995) for unconditional approach. The ability of a given model is tested for specified time horizon of out of sample forecast against actual data. Unconditional methods count the number of exceptions or violations, in point where the realised returns (loss) fall bellow VaR band. Conditional methods tests if the duration time between VaR violations is independent and without the clusters. This test tells us if there are some consecutive exceedances for some time interval. But based on particular null hypothesis of Christoffersen test we can access unconditial property too.

RESULTS

The behaviour of time series of prices during the whole time period shows the non-stationarity of all four time series. This assumption is proven via both KPSS and ADF tests. After the calculation of logarithmical difference, the estimate is performed via ARMA-GARCH model for approximately one hundred and fifty combinations of settings of models. The testing of applicability and selection of applicable models via minimal values AIC a BIC points to several common phenomena. The value of the information criteria is fundamentally influenced by the distribution of the variance process. From this point of view, the highest quality models contain errors with ged, Student-t or Normal Inverse Gaussian distribution. Otherwise, the normal distribution cannot be preferred in statistical reasoning for any of the tested models. Moreover, the testing of multivariate normality was performed. The following conclusion was reached via the test based on Mahalonobis distance and Jarque-Bera test: equity returns ain't been the realisations of the Gaussian distribution. Various combinations of the values p and q in GARCH (p, q) model have only minor influence on the values of information criteria – in the order of magnitude of units of percents. The partial changes in parameters values have mostly impact on the t-values and the full statistical verification of tested model. It is possible to affirm that the model using *p* and *q* equal to one is sufficient for our purposes.

The most acceptable models through in sample testing of unique time series for in sample period are ARMA(3,1)-GJR-ged (Telefónica), ARMA(1,0)-GARCH(1,1)-Student-t (ČEZ), ARMA(1,1)-GJR-Student-t (KB), ARMA(1,0)-GARCH(1,1)-GJR-ged (ERSTE). We have performed almost 150 model verifications, for different error distributions and up to 5th lag order of mean process. With regard to these results, the model ARMA(1,1)-GARCH(1,1)-GJR-Student-t was chosen for further modeling of one-dimensional series of recalculated yields of portfolio. In case of multidimensional models of volatility, the following specifications serve as the basis: VAR(1)-GO-GARCH(1,1)-GJR and VAR(1)-DCC(1,1)-GARCH(1,1)-GJR with multivariate Student-t distribution. In the next step we let the forecast roll over the out of sample period, from



2: *Empirical scatter plots of residuals from initial ARMA filtering for capturing in sample tail dependence patterns and copula types (700 observations within January 2010 and September 2012)*

which the magnitude VaR on 95% confidence level is derived, see Fig. 3 bellow. For the better readability we will use just model notations without process orders.

Estimation of D-Vine Copula Models

For the Vine copula function estimates, first the dependence between magnitudes and the character of their distribution must be clarified and specified in detail. Tighter positive dependence is obvious especially for banking segment, i.e. within one sector. Bolder points on the margins of graphs – lower and upper quantiles – result from bold edges of realized empirical yields. These facts lead to preliminary determination of the appropriate copula function. The cognizance of location of dependence within the multivariate distribution function is essential for the purpose of measurement of the risk of extreme losses. Fig. 2 shows that leading dependences for vines will determine the yields of bank institutions (most intensive dependence), which can be figured appropriately via elliptical Student-t copula, or Archimedean copulas: Gumbel or Clayton – probably without rotation.

From a bivariate exploratory data analysis and the statistical inference procedures we have drawn more specific conclusions. From the results of testing of partial models based on the Vuong-Clark test, the most appropriate categories of models are chosen for the order of dependences: 1st ERSTE, 2nd KB, 3rd ČEZ and 4th Telefónica. Further testing provides us with the families of the copula models, which expresses this structure of dependence in the most appropriate way: we should deal with the Clayton, Gumbel, BB1 or Student-t copulas families. In the next step we use joint MLE procedure for the copula model parameters estimation. According to the estimates of the pair dependencies and their log-likelihood evaluation we offer final results in Tab. II also with above mentioned copulas[5] for the best and two other possible dependence structures (which were not used for simulation). After that we let the D-Vine algorithm separate the residuals into 3 levels, which looks similar to D-Vine constructions in Fig. 1. We recognized from “upper” into “bottom level” dependence intensity for these copula pairs (number quantifies Kendall’s tau correlation coefficient):

1st Level:

- ERSTE/KB – Student-t, 0.43.
- ERSTE /Telefónica – Student-t, 0.23.
- ČEZ/Telefónica – BB1, 0.41.

2nd Level:

- Telefónica, ČEZ copula bounds to ERSTE/Telefónica copula, under global Gumbel copula, 0.23.
- KB, ERSTE copula bounds to ERSTE/Telefónica copula – Student-t, 0.14.

3rd Level:

- KB, Telefónica|ERSTE bound to ERSTE, ČEZ|Telefónica – Frank copula, 0.14.

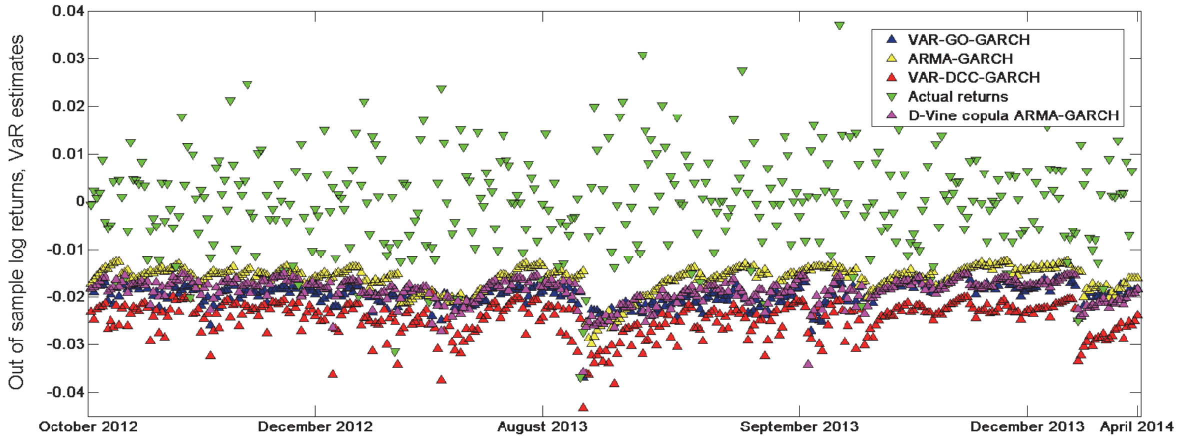
5 Number of copulas is determined from **dimension·(dimension-1)/2**, where dimension is the number of returns vectors. That means that algorithm works with 6 copulas.

II: *Best fitting one or two parametric copulas for residuals D-Vine dependence structure ordered by log-likelihood values (the 1st item is best according to intensity of dependence)*

Ranked dependence	Log-lik	Student-t	Student-t	BB1	Student-t	Gumbel	Frank
1. Erste, KB, Čez, Telefónica	257.87	(0.62, 13.63)	(0.52, 5.49)	(0.21,1.41)	(0.30, 12.13)	(0.15,0)	(0.89, 0)
2. Telefónica, Čez, Erste, KB	257.41	(0.50, 4.70)	(0.51, 6.35)	(0.35,1.48)	(0.23, 11.17)	(1.24,0)	(0.75, 0)
3. Erste, Čez, KB, Telefónica	256.89	(0.52, 7.19)	(0.50, 4.70)	(0.50, 4.70)	(0.50, 4.70)	(0.50, 4.70)	(0.50, 4.70)

III: *Results from backtesting for out of sample period and 18 expected exceedances (from October 2012 to April 2014)*[6]

Statistics	VAR-GO-GARCH	ARMA-GARCH	VAR-DCC-GARCH	D-Vine copula A-G
Realized exceedance	10	11	2	10
Kupiec test: p-value	0.027	0.055	0.00065	0.027
Christoffersen test: p-value	0.047	0.11	0.00046	0.047



3: *Equally-weighted portfolio VaR (at 95% confidence level for a lower tail of the returns distribution) for 1 day ahead out of sample forecast*

From this multivariate copula we generate simulated uniformly distributed numbers [0, 1] for transformation with inverse cdf and finally calculate portfolio returns and VaR at 95% confidence level, see Tab. III and Fig. 3 for visual evaluation.

Forecasting and Backtesting Results

For details about the VaR exceedance and visual fit to empiric returns with backtesting results, see Fig. 3 and Tab. III. According to the Kupiec and Christoffersen test the best model for fitting actual risk levels is simple ARMA-GARCH with 11 violations. Although the Kupiec test does not reject ARMA-GARCH, its numerical values are under the coverage rate (11 actual < 18 theoretical violations like 5% from 370) for 95% confidence interval.

D-Vine copula ARMA-GARCH model overestimates the risk level as well, when its values are on similar level with VAR-GO-GARCH estimates. DCC-GARCH based model performs very poorly in comparison with other models what is also obvious from Fig. 3. Each of VaR lines visualizes the potential 1 day loss on 95% confidence level. If the actual return (green triangle) drops below the VaR forecast band, violation (exceedance) occured. VAR-DCC-GARCH is probably unable to assess correctly the dynamical dependence structure between the residual components in this case and the estimates are more volatile than in other cases.

Fashion of the VaR exceedances across time was tested with Christofferesen test and as in our case with unconditional test hypothesis as well. After hypothesis testing we know that there exists only one suitable modelling approach for selected confidence level: ARMA-GARCH method for univariate time series of equally-weighted portfolio value. It has generated exceedances without clusters in time.

DISCUSSION

Results are at first sight in concordance with results from Bauwens, Laurent & Rombouts (2006), where univariate model offers better forecasting ability than multivariate. D-Vines were not used for

6 Hypotesis of tests: H0 for Kupiec test: Correct number of exceedances H0 for Christoffersen test: Idendenpendence of exceedance occurence and also correct number of exceedances.

VaR forecasting comparison study yet, nor for Czech listed companies, so we have only limited ability to compare results. The main contribution lies in concerning statistically evaluated distributions – Gaussian based models performs to larger extent poorly, mainly due to the fact, the returns are not coming from Gaussian probability distributions. For multivariate volatility models SW packages offer only limited range of multivariate distributions different to Gaussian, Student-t or NIG distribution. So it is often acceptable to use multivariate Student-t distributions.

Copula functions offer many technological possibilities, in the cases of vines with flexible structures for non-symetric tail dependence measurement and visualizations. But in our opinion its results are heavily biased due to frequency of the model recalibration. We practically used only static dependence which was established before the first moving window fit. At least, SW packages offer many possibilities for evaluating of actual risks level, i.e. usual GARCH-copula from Patton (2006).

But so far from the actual results we in general propose ARMA-GARCH-GJR with Student-t, ged or NIG distribution as appropriate way to forecast portfolio VaR when we won't work with dependence between return streams. After the standard backtesting we can take that as possible fact. But obviously the obtained results could be further enhanced via testing of accuracy on different levels of significance.

CONCLUSION

This contribution deals with the application of volatility models and its backtesting when estimating the equity portfolio Value at Risk based on the data of companies ČEZ, a. s., Komerční banka, a. s., ERSTE Bank, a.s, Telefónica Czech Republic from January 2010 to April 2014. The variant comparison of approx. one hundred and fifty combinations of univariate ARMA-GARCH models, multivariate models VAR-DCC-GARCH, VAR-GO-GARCH and D-Vine copula ARMA-GARCH approach led us to the conclusion that the most simple ARMA-GARCH (for modelling of univariate time series as calculated value of the portfolio) fits the data well graphically, especially for the formation of one day ahead estimates in the out of sample time period from October 2012 to April 2014.

The results according to backtesting indicate the overestimation (theoretical coverage is above the actual exceedance of losses) of the risk level for almost all of the model approaches, with exception of ARMA-GARCH model. Kupiec and Christoffersen test proposed that the model is efficient in forecasting VaR. This model is also the only case of model in which the violations are independently distributed in time. The outputs of this research can be further developed via testing of accuracy on different levels of significance, with the use of other backtesting approaches or the use of the C-Vine copula GARCH model when modeling the dependence of market index and stocks from this submarket.

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