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DECISION-MAKING ON IMPLEMENTATION OF IPO UNDER TOPOLOGICAL UNCERTAINTY

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Abstract

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IPO (Initial Public Offering) is a complex decision making task which is always associated with different types of uncertainty. Poor accuracies of available probabilities of lotteries e.g. quantification of investor interest is studied in the first part of this paper (Meluzín, Doubravský, Dohnal, 2012). However, IPO is often prohibitively ill-known. This paper takes into consideration the fact that decision makers cannot specify the structure/topology of the relevant decision tree. It means that one IPO task is specified by several (partially) different decision trees which comes from different sources e.g. from different teams of decision makers/experts. A flexible integration of those trees is based on fuzzy logic using the reconciliation (Meluzín, Doubravský, Dohnal, 2012). The developed algorithm is demonstrated by a case study which is presented in details. The IPO case integrates two partially different decision trees.

Keywords: IPO, decision-making, uncertainty, linear programming, fuzzy logic

INTRODUCTION

This article is an extension of the article (Meluzín, Doubravský, Dohnal, 2012). It is therefore highly desirable to study this paper first. All IPO decisions are inevitably based an inaccurate data, e.g. probabilities, penalties. These inaccuracies can be taken into consideration by the methods shown in (Meluzín, Doubravský, Dohnal, 2012). However, the topology of a decision tree under study can be uncertain as well. To solve IPO decisions, taking into consideration the fact that a team of experts cannot agree of the structure / topology of the decision tree itself (Apostolakis, Lee, 1997), needs more flexible formal tool and cannot be solve just by a single tree reconciliation given in (Meluzín, Doubravský, Dohnal, 2012).

Statement of a Problem

Decision tasks are usually represented by single root trees and sets of available III (input information items), e.g. probabilities, penalties etc. In the practical situations the full III set is either not available or some of its elements are prohibitively vague under realistic conditions, see e.g. (Danielson, Ekenberg, Larsson, 2007), (Nie, Zhang, Liu, 2009). Therefore a methodology is needed to quantify the missing set of III. This problem is described in (Meluzín, Doubravský, Dohnal, 2012) where there is considered strong analogy between a water flow through a one root tree system of pipes and the decision tree of the same topology. This analogy is based on the assumption “The longer the path the less probable the path is”, see e.g. (Dohnal, Vykydal, Kvapilik, 1992). Let us imagine a situation where a firm has incomplete information obtained from different experts about implementation of IPO. This can have two basic consequences. The first consequence may be the existence of one decision tree with different valuations of the same branches e.g. lotteries. The second one may be the existence of topologically different decision trees. It means that two or more decision trees must be used to solve the same decision problem. This problem is solved in the next section as the multi reconciliation.

Multi Reconciliation

The problems of reconciliation are very important and have been studied for more than 30 years, see e.g. (Watson, 1994). Let us suppose several experts are involved in decision making. Then we usually obtained several possible incomplete sets of probabilities (eq. (14) in (Meluzín, Doubravský, Dohnal, 2012)).

$$\mathbf{M} \equiv \{\tilde{\mathbf{R}}_1, \tilde{\mathbf{R}}_2, \ldots, \tilde{\mathbf{R}}_s\}. \quad (1)$$

Each set of probabilities $\tilde{\mathbf{R}}_i$ is a part of a problem Ω_i to generate the corresponding fuzzy set of linear equation (eq. (18) in (Meluzín, Doubravský, Dohnal, 2012)). That means the reconciliation can be solved by a fuzzy linear programming, see e.g. (Kikuchi, 2000), (Fedrizzi, Kacprzyk, Verdegay, 1991). The problem Ω_i is represented by a set of linear equations which consists of a set of balance equations (eq. (10) in (Meluzín, Doubravský, Dohnal, 2012)) and a set of additional probabilities (eq. (17) in (Meluzín, Doubravský, Dohnal, 2012)).

$$\mathbf{\Omega}_i = (\mathbf{Ap})_i = \mathbf{b}_i \cup \mathbf{p}_i = \tilde{\mathbf{R}}_i, \; i = 1, 2, \ldots, s. \quad (2)$$

There are s incomplete sets of probabilities, see (Meluzín, Doubravský, Dohnal, 2012). Therefore there are s problems Ω_i, $i = 1, 2, \ldots, s$ which must be solved. To simplify the problem, let us suppose that the problem Ω is a set of equations. It means that the following set of equations must be solved:

$$\mathbf{\Omega} = (\mathbf{\Omega}_1 \cup \mathbf{\Omega}_2 \cup \ldots \cup \mathbf{\Omega}_s), \quad (3)$$

j-th fuzzy equation $p_j = \tilde{\mathbf{R}}_j$ (eq. (20) in (Meluzín, Doubravský, Dohnal, 2012)) of set of equations Ω_i can be transformed into four linear inequalities (eq. (21)–(24) in (Meluzín, Doubravský, Dohnal, 2012)), see (Tan, Briones, Culaba, 2007). The system of linear equation Ω_i (eq. (18) in (Meluzín, Doubravský, Dohnal, 2012)) is over specified system of linear equation. This system of linear equation can be solved by method of linear programming, see (Huang, Moore, 1993). Just need to introduce an objective function Q_i (eq. (26) in (Meluzín, Doubravský, Dohnal, 2012)).

If Ω represents the well overestimated set of linear equations then this system can be solved by method of linear programming, see (Lai, Hwang, 1992), (Huang, Moore, 1993), with following objective function

$$Q = w_1 Q_1 + w_2 Q_2 + \ldots + w_s Q_s, \quad (4)$$

where $w_1, w_2, \ldots, w_s$ represent the weights assigned to each expert/decision maker. The reconciliation problem described in (eq. (18) in (Meluzín, Doubravský, Dohnal, 2012)) is simply extended as follows:

$$\begin{aligned} &\min Q \\ &\text{s. t.} \; (\mathbf{Ap})_1 = \mathbf{b}_1 \\ &\quad \mathbf{p}_1 = \tilde{\mathbf{R}}_1 \\ &\quad \vdots \\ &\quad (\mathbf{Ap})_s = \mathbf{b}_s \\ &\quad \mathbf{p}_s = \tilde{\mathbf{R}}_s \end{aligned} \quad (5)$$

Case Study

The following factors are taken into consideration to simplify the IPO task:

- Macro-economic growth (increments of GDP);
- Investor's interest in the IPO;
- Size of the issue (number of issued shares multiplied by their emission rate).

The case study decision-making tree is seen in Fig. 1. There are two types of nodes: lotteries – circles and decision-making – squares. A circle represents a decision which is not done by the decision maker. It could be e.g. a market conditions.

Let us suppose that two different trees are presented by two experts, see Fig. 1 and Fig. 3. The reconciliation algorithm, described in (eq. (18) in (Meluzín, Doubravský, Dohnal, 2012)), is used for both trees. The results are presented in the following sub chapters. An attempt is made to integrate these two trees and perform the reconciliation as described by (5). The result is presented in the sub chapter "Integrated estimates of both experts".

Analysis of the Decision Tree Given in Fig. 1 (Expert I)

The reconciliation algorithm used in this subchapter is fully described in (eq. (18), (21)–(26) in

I: *Nodes Descriptions*

Node No. Fig. 1	Meaning	Node No. Fig. 1	Meaning
1	Implement IPO?	15	Size of the IPO
2	STOP	16	Investor's interest
3	GDP	20	Investor's interest
4	STOP	24	Investor's interest
5	Size of the IPO		
6	STOP		
7	Investor's interest		
11	Investor's interest		

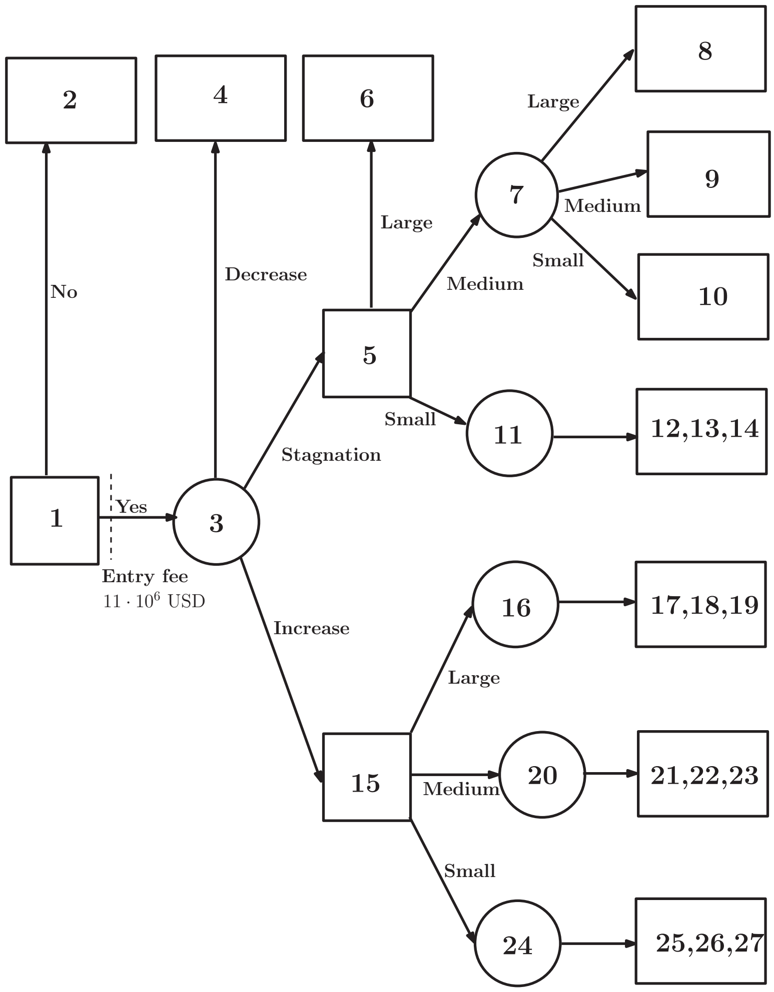
Source: own processing

(Meluzín, Doubravský, Dohnal, 2012)). The decision tree in Fig. 1 is studied. The tree's nodes are characterised in Tab. I.

To make the Fig. 1 easy to present identical squares are integrated into one square. The squares No. 8, 9 and 10 are presented as they are treated during the reconciliation procedure. However, the squares No. 12–14, 17–19, 21–23, 25–27 are graphically presented as one square but treated in the same way as squares 8–10. It means that the lottery No. 11 has three outgoing branches equally as the lottery No. 7, Fig. 1.

The Tab. II shows the inaccurately known probabilities and the profits for all terminals.

From the Tab. II it is obvious, that only some branch probabilities of the corresponding lotteries

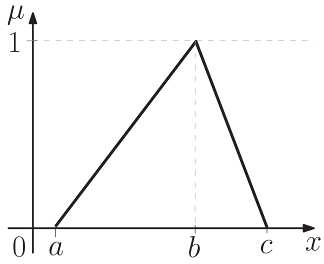


1: *Decision-making tree IPO by first expert*
Source: own processing

II: *Profit and the known probabilities*

Branch	Probability of the variant	Profit (mil. USD)	Branch	Probability of the variant	Profit (mil. USD)
3–4	unknown	0	16–17	unknown	100
3–5	about 0.5	unknown	16–18	unknown	50
3–15	unknown	unknown	16–19	unknown	10
7–8	unknown	30	20–21	unknown	40
7–9	unknown	10	20–22	unknown	20
7–10	unknown	–5	20–23	unknown	0
11–12	about 0.6	10	24–25	unknown	30
11–13	about 0.15	–5	24–26	unknown	20
11–14	about 0.15	–20	24–27	unknown	–10

Source: own processing



2: *Triangular fuzzy number*
Source: own processing

III: *Conversion of probabilities to fuzzy*

Branch Probability	a	b	c
$a_{3,5}$ =	0.3	0.5	0.55
$a_{11,12}$ =	0.5	0.6	0.7
$a_{11,13}$ =	0.1	0.15	0.3
$a_{11,14}$ =	0.1	0.15	0.3

Source: own processing

are known. The evaluation of the unknown probabilities is based on metaheuristics (3) in (Meluzín, Doubravský, Dohnal, 2012). The known fuzzy probabilities are taken into consideration as triangular fuzzy numbers, see Fig. 2.

Tab. III gives the full description of all fuzzy numbers; see Fig. 2 and Tab. II.

The remaining probabilities are calculated on the basis of the topological resistance, balancing equations and the partial ignorance (eq. (7), (10), (15) in (Meluzín, Doubravský, Dohnal, 2012)), see the following Tab. IV.

The calculated expected profits for individual sub decisions are given in Tab. V. For example, the choice of the best branch from the following branches 5–6, 5–7 and 5–11 needs requires the evaluation of the following lotteries: 7 and 11. The expected value of the lottery 7 is simple as all the relevant probabilities are the same namely 0.333, see Tab. IV. The profits reached by the terminals No. 8, 9 and 10 are 30, 10 and –5, see Tab. 2. The values of all lotteries are given in Tab. V.

The lottery 7 has the value 11.66 and the lottery 11 has the value –4.995. The terminal No. 6, see Fig. 1, has the profit, see Tab. II, equal to 0. Therefore the maximum value from terminal No. 7 is 11.66.

Tree Evaluation is done in the following steps:

If the entry fee is 11 mil. USD, then the following are optimal decisions:

1) Implement the IPO in case of stagnation, or of GDP growth, see the branch 3–5, 3–15 Fig. 1.

IV: *The splitting ratios*

Branch	Calculated Probability	Branch	Calculated Probability
3–4	0.271	16–17	0.333
3–5	0.500	16–18	0.333
3–15	0.229	16–19	0.333
7–8	0.333	20–21	0.333
7–9	0.333	20–22	0.333
7–10	0.333	20–23	0.333
11–12	0.700	24–25	0.333
11–13	0.150	24–26	0.333
11–14	0.150	24–27	0.333

Source: own processing

V: *Expected lottery value*

Branch	5–7	5–11	15–16	15–20	15–24	1–3	1–2
Expected lottery value (mil. USD)	11.66	–4.995	53.28	19.98	13.32	18.03	0

Source: own processing

VI: *Characterisation of Nodes*

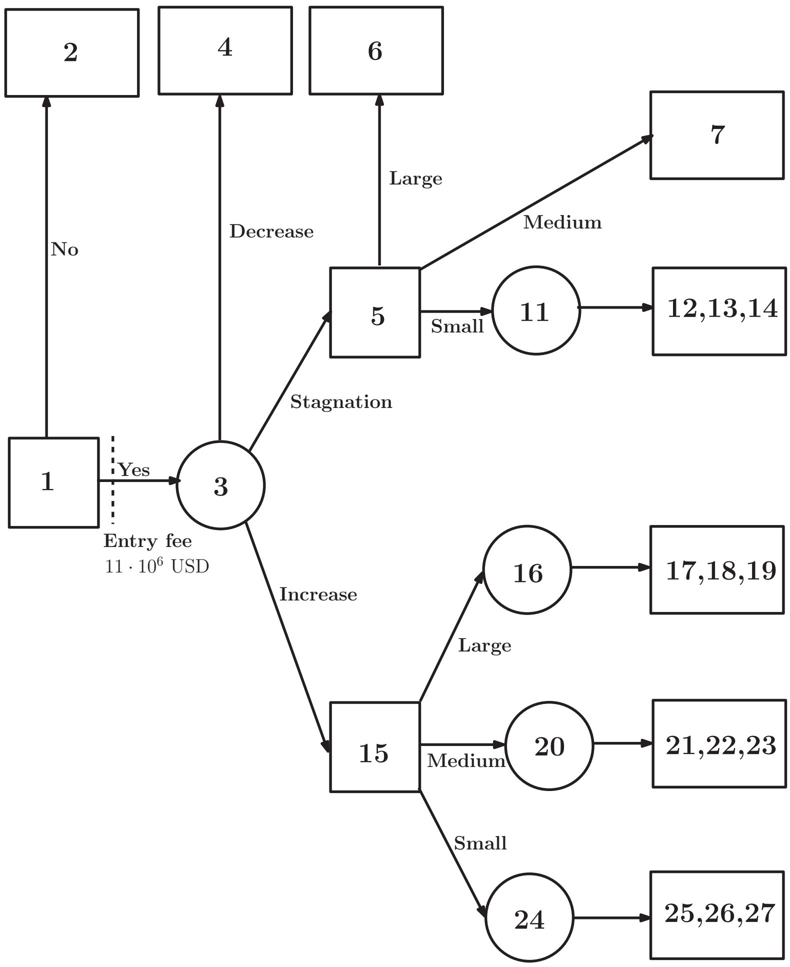
Node No. Fig. 3	Meaning	Node No. Fig. 3	Meaning
1	Implement the IPO?	11	Investor's interest
2	STOP	15	Size of the IPO
3	GDP	16	Investor's interest
4	STOP	20	Investor's interest
5	Size of the IPO	24	Investor's interest
6	STOP		
7	STOP		

Source: own processing

VII: *Profit and the specified probabilities*

Variant	Probability of the branch	Profit (mil. USD)	Variant	Probability of the branch	Profit (mil. USD)
3–4	unknown	0	20–21	unknown	40
3–5	about 0.5	unknown	20–22	unknown	20
3–15	unknown	unknown	20–23	unknown	0
11–12	about 0.6	10	24–25	unknown	30
11–13	about 0.15	–5	24–26	unknown	20
11–14	about 0.15	–20	24–27	unknown	–10
16–17	unknown	100			
16–18	unknown	50			
16–19	unknown	10			

Source: own processing



3: *Decision-making tree IPO by the second expert*
Source: own processing

2) In case of stagnation of GDP choose middle issue, see the branch 3–5–7, in the case of GDP growth choose a big issue, see the branch 3–15–16, Fig. 1.
3) Highest profit will be achieved in the case of great interest of investors in both variants.

Expected value of profit is USD 7.03 mil. USD.

Analysis of the Decision Tree Given in Fig. 3 (Expert II)

Fig. 3 is the second studied decision tree, see Tab. VI.

The only difference between two trees, see Fig. 1 and 3 is the branch 5–7. Tab. VII shows the profits of the individual variants of the known probabilities obtained from the second expert.

VIII: *Conversion of probabilities to fuzzy*

Branch Probability	a	b	c
$a_{3,5}$ =	0.25	0.5	0.6
$a_{11,12}$ =	0.5	0.6	0.7
$a_{11,13}$ =	0.1	0.15	0.3
$a_{11,14}$ =	0.1	0.15	0.3

Source: own processing

IX: *The splitting ratios*

Branch	Probability	Branch	Probability
3–4	0.310	16–19	0.333
3–5	0.500	20–21	0.333
3–15	0.190	20–22	0.333
11–12	0.600	20–23	0.333
11–13	0.150	24–25	0.333
11–14	0.250	24–26	0.333
16–17	0.333	24–27	0.333
16–18	0.333		

Source: own processing

X: *Expected lottery value*

Branch	5–7	5–11	15–16	15–20	15–24	1–3	1–2
Expected lottery value (mil. USD)	0	0.25	53.28	19.98	13.32	12.33	0

Source: own processing

XI: *Weights*

c	1	2	3	4	5	6	7	8	9	10	11
w_1	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100	0.000
w_2	0.000	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	1.000

Source: own processing

The Tab. VIII shows the relevant entered probabilities obtained of second expert (expert II). The given/known probabilities are again quantified as fuzzy numbers; see Fig. 3, in Tab. VIII.

The remaining probabilities are calculated on the basis of topological resistance, balancing equations and the partial ignorance, see Tab. IX.

The resulting profits are, see Tab. X.

Tree evaluation, see Fig. 3 – If the entry fee is 11 million USD, then in the present case, it is recommended not to realise the IPO, since the expected value of the profit results is –0.752 mil. USD.

Integrated Estimates of Both Experts

It is clear that an integrated reconciliation based on the complex formula (5) requires evaluations of some sort of weights *w* of both experts. It is fully understandable that such evaluation is very subjective. To minimise the relevant risks several weights are studied, see Tab. XI.

The first pair of the weights indicates clearly that the second expert has zero weight, see Tab. XI. The reconciliation problems No. 1 and 11 can be solved using algorithm given in (Meluzín, Doubravský, Dohnal, 2012), see the previous subchapters. Using the formula (5) the integrated reconciliation problems are solve for pairs of weights No. 2–10, see Tab. XI. The resulting splitting fractions are given in Tab. XII.

There are 26 branches in Tab. XII, see Fig. 1. However, the splitting ratios are the same for rows No. 4, 9–26. The splitting ratios for branches 1–4 and 6–8 are not the same. It is easy to identify that there are three subsets of the columns which have different values of the splitting ratios. The first column is unique. The columns 2–5 have the same splitting ratios and the remaining columns 6–11 represent the third set. The corresponding objective functions Q (5) are given in Tab. XIII.

On the basis of these data sets, see Tab. XII, profits for individual variants are calculated. The results are given in Tab. XIV.

Integrated Evaluation done in the following steps:

If the entry fee is 11 million USD, then from these calculations is obvious that in the present case, it is recommended:

1) Implement the IPO in case of stagnation, or of GDP growth, see the branch 3–5, 3–15 Fig. 3.
2) In the case of stagnation of GDP choose middle issue, see the branch 3–5–11, in the case of GDP growth choose a big issue, see the branch 3–15–16, Fig. 3.
3) Highest profit will be achieved in the case of great interest of investors in both variants.

Expected value of profit is USD 1.326 mil. USD.

XII: *The splitting ratios*

r	Branch	Probability										
		1	2	3	4	5	6	7	8	9	10	11
1	1–2	0.962	0.957	0.957	0.957	0.957	0.957	0.957	0.957	0.957	0.957	0.957
2	1–3	0.038	0.043	0.043	0.043	0.043	0.043	0.043	0.043	0.043	0.043	0.043
3	3–4	0.271	0.271	0.271	0.271	0.271	0.310	0.310	0.310	0.310	0.310	0.310
4	3–5	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5	3–15	0.229	0.229	0.229	0.229	0.229	0.190	0.190	0.190	0.190	0.190	0.190
6	5–6	0.444	0.444	0.444	0.444	0.444	0.417	0.417	0.417	0.417	0.417	0.417
7	5–7	0.278	0.278	0.278	0.278	0.278	0.305	0.416	0.416	0.416	0.416	0.416
8	5–11	0.278	0.278	0.278	0.278	0.278	0.278	0.167	0.167	0.167	0.167	0.167
9	7–8	0.333	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	7–9	0.333	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	7–10	0.333	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	11–12	0.700	0.600	0.600	0.600	0.600	0.600	0.600	0.600	0.600	0.600	0.600
13	11–13	0.150	0.250	0.250	0.250	0.250	0.150	0.150	0.150	0.150	0.150	0.150
14	11–14	0.150	0.150	0.150	0.150	0.150	0.250	0.250	0.250	0.250	0.250	0.250
15	15–16	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333
16	15–20	0.334	0.334	0.334	0.334	0.334	0.334	0.334	0.334	0.334	0.334	0.334
17	15–24	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333
18	16–17	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333
19	16–18	0.334	0.334	0.334	0.334	0.334	0.334	0.334	0.334	0.334	0.334	0.334
20	16–19	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333
21	20–21	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333
22	20–22	0.334	0.334	0.334	0.334	0.334	0.334	0.334	0.334	0.334	0.334	0.334
23	20–23	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333
24	24–25	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333
25	24–26	0.334	0.334	0.334	0.334	0.334	0.334	0.334	0.334	0.334	0.334	0.334
26	24–27	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333

Source: own processing

XIII: *Value of objective function (5)*

c	1	2	3	4	5	6	7	8	9	10	11
Q	4.107	3.872	3.627	3.381	3.136	2.864	2.557	2.174	1.791	1.407	1.024

Source: own processing

XIV: *Expected lottery value*

Variant	5–7	5–11	15–16	15–20	15–24	1–3	1–2
Expected lottery value (mil. USD)	0	0.25	53.28	19.98	13.32	12.326	0

Source: own processing

CONCLUSION

The paper solves probably the most difficult IPO problem related to uncertainties of decision making trees, namely topological uncertainties taking into consideration missing values of probabilities of some outcomes of lotteries etc. However, a risk aversion is not incorporated into the described decision making algorithm. It is a serious disadvantage of the presented algorithms.

It is a well-known fact that if significant losses are possible then the evaluation of lotteries are not based on the expected values as it is done in this paper. An attempt will be made to take some risk aversions into consideration. However, it means that a nonlinear element will be considered. Simple common sense reasoning can easily discover that if the risk increases that the evaluation of lotteries is more and more pessimistic. It means that there is an unknown upper value which a decision maker is ready to risk irrespective of potential profits.

The described algorithm can be used not just for IPO problems. Its spectrum of different decision tasks is relatively broad ranging from e.g. ecological aspects of investments to complex engineering/economics/social tasks.

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