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CONTRIBUTION OF SIMPLE HEURISTICS FOR THE VEHICLE ROUTING PROBLEM – A CASE STUDY OF A SMALL BREWERY

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Abstract

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This paper presents a case study of a local brewery situated near to Prague. Even though its management already has the software which solves the vehicle routing problem by Mayer and Branch and Bound methods, it is still favourable to implement more approximation methods. The basic reason is that the Branch and Bound algorithm is complicated and software overflows may occur during its run in case of more cities in the cycle. The aim of this paper is to solve the real transportations using other methods that satisfy basic requirements of practicability.
Transportation, which the brewery carried out during the selected week, provided data which were then analyzed using both the modifications of "classical" approximation methods (such as by Clark and Wright, Habr, and Mayer). Three types of the combination of these methods were also applied. The computed results were compared with the routes actually used by the brewery and with results from already implemented software. They showed that the brewery can save approximately 50,000 CZK (2,000 EUR) per year. Furthermore, the application of these methods needs neither any special technical equipment nor much time for the computation.

vehicle routing problem, case study, Mayer method, savings method, Habr frequencies, methods crossing

The optimization problem of the delivery of a specific material, where a round trip is realized, can in reality be encountered with very often. It means that one central and a certain number of other places (cities, points …) are given. The vehicles start in the central city, each of the other cities is visited by at least (or just, according to the particular type of the problem) one vehicle and the vehicles usually come back to the central city (in this case, the task is called close; otherwise, it is called open). The case when the transportation among all serviced places is to be realized by one circle is named Traveling Salesman Problem (TSP). The case when more vehicles are necessary, e.g. due to capacity or time reasons, is named Vehicle Routing Problem (VRP). Both these cases belong among the NP-hard problems. This type of tasks is distinguished by a non-existent efficient algorithm which would be able to find their precise theoretical optimum. However, for such types of tasks so-called heuristics (approximation methods) are created, offering only approximate solutions, however, of so high-quality that we can regard them as economic optimums. Beside this, it is possible to use the Branch and Bound method, which can find the precise optimum but its computation time grows exponentially with data size and the tasks of size about 20 cities may not be solvable in this way using common computers.

One of the first heuristics for TSP and VRP is the savings method by Clarke and Wright (1964). Also Habr (1966), Czech scientist, regarded as a founder of the Czech systems science school, is the author of several other early heuristics based on so-called frequencies. These methods can be easily applied even for manual computation for small or medium size tasks. On the other hand, since the

1990's, thanks to information and communication technologies development, much more complex approaches to solving the NP-hard problems are proposed, one of them are genetic algorithms (e.g. Braun (1991) for TSP, Potvin, Dube, Robillard (1996) for VRP, Perzina (2011) for other tasks)[1].

In practice, companies rarely pay much attention to dealing with such problems; especially in case of small or medium size tasks or if transportation is not their main business. Kučera and Jarkovská (2010) demonstrates on a case study of NOPEK Bakery in Vysoké Mýto how the application of different simple approximation methods can decrease the number of vehicles necessary for so-called "fast deliveries" by 18%. Also other case studies on delivery planning have recently been published. In some cases commercial software is used for solution. For example, studies from the central Finland which concern a route proposal for seniors home care (home care nurses, meal delivery or elderly transportation are the VRP) used optimisation heuristics (described in Hasle, Kloster (2007)) from commercial Spider software (Bräysy, Dullaert, Nakari, 2009; Bräysy *et al.*, 2009). Commercial software ROUTER@ was used for oil distribution planning for Petramina company from its depot to gas stations in one Jakarta district (Soehodho, Pramono, 2003). In both cases, significant savings were reached in comparison with the former delivery organization. In other cases, heuristics are used for solution, as for instance in the optimization of the municipal refuse collecting system (Plevný, 2003), in planning lumber haulage (Thiele, 2008) or collections of daily reports from affiliated branches of a bank to headquarters (Pelikán, 2006). In all these three cases, one of the heuristics, which even proved as relatively successful and suitable for this purpose, was also the savings method, i.e. one of the methods used in this paper. Suthikarnnarunai (2008) compares the heuristics application with exact optimum computation using integer programming with the case of the transportation of the University of The Thai Chamber of Commerce employees by university buses to work. While the exact computation brought the result in sensible computational time only in some cases and in others it failed, using the heuristics results were obtained within a small amount of computational time and were good in comparison with the exact computation.

This paper presents a case study of a local brewery situated near to Prague. This VRP is closed (i.e. the vehicles return back to the place where they start from) and limited by the vehicle capacity (while on the other hand, e.g. Kučera (2012a) studies the time limited open VRP), i.e. exactly of the same type as that in Kučera, Houška, Beránková (2006) called Multipletours Traveling Salesman Problem. Some of the methods used here are tested there on randomly generated test cases. Hadrava (2013) uses for the brewery VRP the Mayer method for dividing the customers among single vehicles and then their routes are sorted by the Branch and Bound method (which computes optimum, but only for single sub-cycles chosen by Mayer method). Beside this, Hadrava (2013) focuses on transportation carried out during the selected week and compare the results obtained by the macro with actual routes used by the brewery. For each day the macro improved the original transportation plan.

Discussion with the management showed one important issue – the sustainability of the actual solution. The firm can not invest into professional software but is able to service the VBA macro. Moreover, in case of investment into vehicles with bigger capacity, the higher number of vertices in one cycle could result into fail of Branch and Bound algorithm in the VBA macro. As a result, more heuristics should be implemented.

This paper describes a case study where the application of basic methods brought savings, which are significant for real subject. The aim of this paper is to solve these transportations using other methods presented in the following chapter and, based on the comparison of the obtained results, to propose which of them is most suitable and e.g. should be used in the macro instead of the Mayer method. Our goal is not to introduce fundamentally new or improved methods for TSP or VRP, but to present the impact and benefits of known methods in business practice. The paper describes case study where the application of basic methods brought significant savings and software solution remains comfortable for such a firm.

MATERIALS AND METHODS

According to European Commission's (2013) classification this brewery belongs to micro-enterprises (i.e. the company has less than 10 employees and turnover less than 2 millions EUR). This brewery supplies 73 restaurants and pubs in surrounding area and in Prague. Each day, beer is distributed to approximately 20 of them. The vehicles used for the transportation have the capacity of 20 positions, each position for either 2 large barrels, 3 small barrels, or 1 large and 1 small barrel. The cost matrix has already been imported into Server 2008 R2 Express by Hadrava (2013).

The whole process started with discussion with management of the brewery. During the discussion, the management specified requirements on the mathematical and software solution:

1. Responsible employee of the firm must understand the chosen methods, i.e. the VBA

1 For more detail about TSP see e.g. Applegate *et al.* (2006), Gutin and Punnen (2007); for VRP see e.g. Toth and Vigo (2002).

macro must not be the black box. Commercial VRP specialised software is not suitable for the company.
2. They prefer more methods. Namely, in different situation, different heuristic could produce results of different quality.

As a result, the following methods were chosen.

Used approximation methods

For the description of the chosen methods, let us introduce some basic notation, which is common for all of them. The central city will get index 0 and the other cities numbers from 1 to n. The cost matrix will be denoted by C (consisting of single costs of transportation c_{ij}, $i, j = 0, \ldots, n$).

Mayer method (MM)

MM starts the choice of cities for a particular vehicle tour by the remotest city from the central one. Then it adds other cities in a way similar to the Prim (Jarník) algorithm for the minimum spanning tree as long as the vehicle capacity enables this:

1. The first place inserted into a tour is the place with maximal distance from the central place.
2. The second place inserted into the tour is the place with minimal distance from the first inserted place.
3. Than we chose from the set of places which have not been inserted yet the place with a minimal distance to any already inserted place.
4. Repeat step 3 until the capacity of the vehicle is used up.
5. Start another tour construction with step 1 (using only places not inserted in previous tours).

Note that MM does not solve VRP completely. It only divides the cities into groups for single vehicles. The route for each vehicle must be constructed by some of the methods for TSP. In case of our paper the Branch and Bound method was applied.

Savings method (SM)

This method is based on comparing lengths of the straight route between any two cities and the route via the central city:

1. For all pairs of other cities (i, j) compute the savings $s_{ij} = c_{i0} + c_{0j} - c_{ij}$.
2. Process edges (straight routes between pairs of cities) according to the descending order of the savings s_{ij}. If after adding this edge all the edges so far added to the route form the set of vertex disjoint paths and for each path the sum of the capacities of the cities lying on it does not exceed the capacity of the vehicle, then add it to the solution. Repeat this until each city lies on some of the paths and joining arbitrary two paths the allowed capacity is exceeded.
3. In the end add the city 0 to all the paths to create cyclic routes.

Thus, we use the parallel processing version which is more accurate, but slightly more complex than the sequentially processing one proposed e.g. in Pelikán (2006).

Habr frequencies approach (HFA)

The disadvantage of savings is that they compare a given edge with only one route via only one city chosen for all the computation. Habr (1966) introduced so called frequencies, which compare the edge with all the others, even non-adjacent edges. He applied them in approximation methods for different transportation problems. He designed even several heuristics for TSP and VRP using them.

Habr frequency for the edge is the value

$$F_{ij} = \sum_{k=1}^{n}\sum_{l=1}^{n}(c_{ij} + c_{kl} - c_{il} - c_{kj}) .$$

This form obviously shows its sense. There exists another form, called modified frequency, more suitable for computations:

$$F'_{ij} = c_{ij} - r_i - s_j,$$

where r_i and s_j are the arithmetic means of the costs of i-th row and j-th column of C, respectively. F'_{ij} can be derived from F_{ij} by linear transformation.

Habr frequencies consider all edges with the same importance. But in the case of MTTSP the edges from/to the central city are more important (more frequently and often used) than the others. Now we show how big this difference is: Let us suppose that p vehicles (cycles) will be used for the transportation (p can be estimated e.g. as $[w/v]$, where w is the sum of the capacities of all the cities and v is the capacity of the vehicle). Let us consider a randomly chosen (with an uniform probability distribution, without respect to the costs) solution with p cycles. The probability of the choice of an edge non-incident to the central city is

$$\frac{n-p}{n(n-1)}$$

(we consider directed edges) while for the edges from/to the central city this probability is equal to p/n. So the edges incident to the central city are

$$\frac{pn-p}{n-p}\text{-times}$$

more important than the others (they occur in the solution with

$$\frac{pn-p}{n-p}\text{-times}$$

greater probability). Thus the frequencies for the MTTSP will be computed by the formula

$$F_{ij} = \sum_{k=1}^{n}\sum_{l=1}^{n}(c_{ij} + c_{kl} - c_{il} - c_{kj}) + \\ + \frac{pn-p}{n-p}\sum_{m=1}^{n}(2c_{ij} + c_{m0} - c_{i0} - c_{mj} + c_{0m} - c_{im} - c_{0i}) \quad (1)$$

or modified frequencies F'_{ij} can be computed by a formula derived from (1) by an analogous linear transformation as in the general case above, which we do not mention here.

Now a method based on the Habr frequencies similar to the previous one (based on savings) can be proposed:

1. For all pairs of other cities (i, j) compute the frequencies F_{ij} according to (1).
2. Process edges (straight routes between pairs of cities) according to the ascending order of the frequencies F_{ij}. If after adding this edge all the edges so far added to the route form the set of vertex disjoint paths and for each path the sum of the capacities of the cities lying on it does not exceed the capacity of the vehicle, then add it to the solution. Repeat this until each city lies on some of the paths and joining arbitrary two paths the allowed capacity is exceeded.
3. In the end add the city 0 to all the paths to create cyclic routes.

Crossing SM and HFA

The following approach is motivated by genetic algorithms. The solution by SM and HFA are crossed in the same way as might be in genetic algorithms. The procedure is proposed and tested on some randomly generated test cases in Kučera (2012b).

The savings matrix and frequencies matrix were normalized using the formula

$$r_{ij} = \frac{a_{ij} - D}{H - D}, \quad (2)$$

where R and A are the normalized and original matrices, respectively, and H and D are the best and worst value in the original matrix A. This normalization converts both maximization and minimization criterion to the form where all the edges obtain evaluations between 0 and 1, the best edge gets 1 and the worst one 0.

Then, the following three matrices were created:

- the matrix, elements of which were maxima of corresponding elements of the original (savings and frequencies) matrices,
- the matrix, elements of which were minima of corresponding elements of the original matrices,
- the matrix, elements of which were average values of corresponding elements of the original matrices.

Next, we applied on these three matrices a procedure analogous to SM and HFA. Thus, we obtain three new methods, we will call them dominant crossing (DC), recessive crossing (RC), and average crossing (AC), respectively; and other three solutions.

RESULTS AND DISCUSSION

Tab. I presents total lengths of actual transportations (AT) by the brewery and solutions by the methods presented above in single days during the selected week when the transportation was observed. In all these days, 17 or 18 restaurants and pubs were to be served. For each day, the best result is highlighted. Cost matrices for the tested week are in appendix A. The last column contains the sum of kilometres for the whole week, the last row expresses the difference between the actual day kilometres and the best result obtained by a tested approximation method. The original result by Hadrava (2013) is in the row denoted as MM (the Mayer method plus Branch and Bound).

First of all, all the methods provided significantly better solutions than the original transportation plan by the brewery. The worst result was found by MM on Wednesday but it was still less than 90% of the original length. On the contrary, the best result was achieved by DC on Monday, the identified way of distribution was less than 58% of the original one.

Among the approximation methods, SM did best (most best solutions, lowest sum of all days), surprisingly better than HFA, although the frequencies are created in more sophisticated way than the savings; of course, it confirms a similar result in (Kučera, Houška, Beránková, 2006). The crossings (both DC and RC) also offered sometime the best results which give an idea about the strength of genetic algorithms. The sums of routes lengths during all five days by RC and AC were only by 1.5 and 0.7 kilometres longer than by SM, respectively. Also the difference between SM and DC, which is less than 10 kilometres, is really small (in fact, it was less than 1.5% of the total length by SM). The worst results among the approximation methods were

I: *Results – kilometres according to used method*

	Mon	Tue	Wed	Thu	Fri	Total
AT	232.5	173.7	136	213.4	156.5	912.1
MM	182.1	126.2	121.7	157.8	127.1	714.9
SM	149	**115.8**	**114**	160.9	**119.9**	**659.6**
HFA	149.5	124.6	120.2	181.8	133.3	709.4
DC	**134**	125.1	119.7	170.1	120.3	669.2
RC	146.5	123.3	115.8	**154**	121.5	661.1
AC	134.5	124.8	121.1	159.6	120.3	660.3
Best – AT	98.5	57.9	22	59.4	36.6	252.5

reached by MM (originally implemented in Hadrava (2013)). As the number of vehicle is concerned, on Tuesday two vehicles sufficed and were truly used in all the transportation plans; in the other days three vehicles were necessary and used in all plans.

In case of the presented week, the savings could be nearly 28%. In comparison with the authors referred in Introductiuon, Bräysy, Dullaert and Nakari (2009) reports improvement by 42.7–52% depending on single cases. Soehodho and Pramono (2003) mention savings about 16.35%. In both these situations, the solved problem was much more complicated. Time windows or replenishment thresholds represent significant increase of the number of conditions. Beside this, in both the cases commercial software using more sophisticated heuristics was applied. From the authors applying simple heuristics, Plevný (2003) achieved savings 11% using the sweep algorithm by Gillet and Miller (1974) and 5% by SM. Kučera and Jarkovská (2010) optimized several sectional transportation plans with savings up to 11% by SM and even up to 14% by the nearest neighbour method (i.e. perhaps the simplest possible heuristic) derived from Rosenkrantz, Stearns and Lewis (1977).

Simple comparison of percentage change in different cases is meaningless because of the different quality of original solution and problem complexity. Important message lays in what they have in common – each time the vehicle routing problem is not solved only intuitively, when even simple approximation method is used, the effort is rewarded by significant savings.

CONCLUSIONS

If the brewery uses the presented approximation methods, it can significantly reduce the costs. If we suppose the fuel price of 36 CZK per litre and the consumption of 10 litres per 100 km, the brewery can save 909 CZK in the presented week (see saved kilometres in Table I). As the real situation of the brewery is concerned, the customers' demand differs each week, new customers occur and sometimes the business relations come to end. Nevertheless, the total possible savings reaches about 50,000 CZK (approximately 2,000 EUR) per year.

The paper does not deal with the generalisation of results for different methods. In different situations the results order of single methods could be totally different. The benefit of these approximations is their time cost. Thus, we recommend using all these methods simultaneously and chose the best result, which will fit the current situation. As a result, the VBA macro in Excel presented in Hadrava (2013) will be improved and all the chosen methods will be implemented.

Annex A: Cost Matrixes

I: *Monday*

	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
Brewery – Z		8.5	5.5	8.4	11.9	4.1	20	14.5	8.6	4.6	15.7	13.6	15.8	12.7	40.2	20.3	17.5	13.7	16.4
A	8.5		11.6	0.8	4.1	4.9	12.2	22.5	0.2	11.6	7.9	5.8	8	3.7	32.4	12.5	9.9	5.9	8.6
B	5.5	11.6		11.4	15	7.2	23.1	11.8	11.7	1	18.8	16.7	18.9	15.8	43.3	23.8	20.6	16.98	19.4
C	8.4	0.8	11.4		3.6	4.8	11.6	22.4	0.6	11.6	7.4	5.3	7.5	4.4	31.9	12	9.2	5.4	8
D	11.9	4.1	15	3.6		7.3	8.1	24.9	3	15.4	3.9	4.8	3.9	1.9	21.5	10.9	5.6	4.8	4.5
E	4.1	4.9	7.2	4.8	7.3		1.4	18.1	5	7.2	12.1	10	12.2	9.1	32.6	15.9	13.9	10.1	12.8
F	20	12.2	23.1	11.6	8.1	1.4		31.3	10.9	22	6.6	9.3	7.3	7.3	12	14.3	8.8	9.4	5.1
G	14.5	22.5	11.8	22.4	24.9	18.1	31.3		21.9	11.2	31.1	26.9	31.2	29.7	39	38.2	32.9	27	31.8
H	8.6	0.2	11.7	0.6	3	5	10.9	21.9		11.7	7.7	5.6	7.8	3.4	28.1	12.3	9.5	5.7	8.3
I	4.6	11.6	1	11.6	15.4	7.2	22	11.2	11.7		18.8	16.7	18.9	15.8	39.3	23.4	20.6	16.8	19.5
J	15.7	7.9	18.8	7.4	3.9	12.1	6.6	31.1	7.7	18.8		3.2	1.1	3.4	17.9	8.1	2.2	3.3	3
K	13.6	5.8	16.7	5.3	4.8	10	9.3	26.9	5.6	16.7	3.2		2.8	3.1	21.2	6.6	4.3	2.1	6.1
L	15.8	8	18.9	7.5	3.9	12.2	7.3	31.2	7.8	18.9	1.1	2.8		3.6	18.5	7.6	2.3	2.8	3.5
M	12.7	3.7	15.8	4.4	1.9	9.1	7.3	29.7	3.4	15.8	3.4	3.1	3.6		21.8	8.6	5.2	2.8	4.1
N	40.2	32.4	43.3	31.9	21.5	32.6	12	39	28.1	39.3	17.9	21.2	18.5	21.8		25.6	19.6	21.5	16.6
O	20.3	12.5	23.8	12	10.9	15.9	14.3	38.2	12.3	23.4	8.1	6.6	7.6	8.6	25.6		8.7	7.6	10.6
P	17.5	9.9	20.6	9.2	5.6	13.9	8.8	32.9	9.5	20.6	2.2	4.3	2.3	5.2	19.6	8.7		4.2	5.3
Q	13.7	5.9	16.98	5.4	4.8	10.1	9.4	27	5.7	16.8	3.3	2.1	2.8	2.8	21.5	7.6	4.2		6.2
R	16.4	8.6	19.4	8	4.5	12.8	5.1	31.8	8.3	19.5	3	6.1	3.5	4.1	16.6	10.6	5.3	6.2	

II: *Tuesday*

	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
Brewery – Z		5.9	0.7	9.6	15.7	5.6	7.7	16.1	10.9	8.4	9.9	12.5	11.7	11.8	5.1	3	17.7	32.1
A	5.9		5.4	8.7	15.1	7.2	10.7	15.2	10	7.6	9.1	11.7	10.9	10.9	8	8.9	16.8	31.2
B	0.7	5.4		10.2	16.5	6.3	8.1	16.7	11.5	9.1	10.5	13.1	12.3	12.4	3.6	1.5	18.3	32.7
C	9.6	8.7	10.2		8.7	5	15.9	9.5	3.7	1.4	3.1	5.9	5.1	4.6	12.2	10.1	11.1	24.9
D	15.7	15.1	16.5	8.7		11	22	8.8	5.7	8.1	7.4	8.4	8.5	6.2	18.2	16.2	12.5	17.4
E	5.6	7.2	6.3	5	11		11	11.3	6.1	3.7	5.2	7.8	5.8	7	7.3	5.3	12.9	27.3
F	7.7	10.7	8.1	15.9	22	11		22.3	17.2	14.6	16.1	18.7	17.9	18.1	3.7	5.8	23.9	38.7
G	16.1	15.2	16.7	9.5	8.8	11.3	22.3		6.7	7.1	6.4	4.3	5.9	7.2	17.3	15.3	7.6	15.1
H	10.9	10	11.5	3.7	5.7	6.1	17.2	6.7		3.1	2.4	4.3	3.5	1.7	13.2	11.2	9.5	21.7
I	8.4	7.6	9.1	1.4	8.1	3.7	14.6	7.1	3.1		1.6	4.5	3.7	3.8	10.7	8.7	9.7	20
J	9.9	9.1	10.5	3.1	7.4	5.2	16.1	6.4	2.4	1.6		2.9	2.1	4	12	9.9	8.1	18.3
K	12.5	11.7	13.1	5.9	8.4	7.8	18.7	4.3	4.3	4.5	2.9		1.7	6.1	13.8	11.8	6.2	15.7
L	11.7	10.9	12.3	5.1	8.5	5.8	17.9	5.9	3.5	3.7	2.1	1.7		5.7	13.7	11.7	7.1	16.7
M	11.8	10.9	12.4	4.6	6.2	7	18.1	7.2	1.7	3.8	4	6.1	5.7		14.3	12.3	10.5	18.6
N	5.1	8	3.6	12.2	18.2	7.3	3.7	17.3	13.2	10.7	12	13.8	13.7	14.3		2.1	20.2	30.5
O	3	8.9	1.5	10.1	16.2	5.3	5.8	15.3	11.2	8.7	9.9	11.8	11.7	12.3	2.1		18.1	32.5
P	17.7	16.8	18.3	11.1	12.5	12.9	23.9	7.6	9.5	9.7	8.1	6.2	7.1	10.5	20.2	18.1		13.3
Q	32.1	31.2	32.7	24.9	17.4	27.3	38.7	15.1	21.7	20	18.3	15.7	16.7	18.6	30.5	32.5	13.3	

III: *Wednesday*

	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
Brewery – Z		3.1	3.8	0.7	5.7	5.6	14.1	8.4	4.1	9.6	9.4	16.5	14.7	16.5	15.6	14.1	14.1	21.6
A	3.1		6.9	3.7	8.8	3.6	15.6	7.2	4.1	9.5	11.9	16.4	14.6	16.4	15.5	14.3	14	21.5
B	3.8	6.9		3.3	1.9	7.4	15	7.7	7.1	9	10.2	15.9	14.1	15.9	14.9	13.8	13.5	21
C	0.7	3.7	3.3		5.2	6.3	16.3	9	5.6	10.3	8.9	17.2	15.4	17.2	16.2	15.1	14.7	22.3
D	5.7	8.8	1.9	5.2		7.8	15.4	8.1	7.4	9.4	12.1	16.3	14.5	16.3	15.3	14.2	13.8	21.4
E	5.6	3.6	7.4	6.3	7.8		10.9	3.6	1.2	4.6	14.4	11.8	10	11.8	10.8	9.7	9.4	16.9
F	14.1	15.6	15	16.3	15.4	10.9		7	10.1	5.6	23.6	1.5	4.9	3.3	0.5	2.2	2.2	8.6
G	8.4	7.2	7.7	9	8.1	3.6	7		4.3	1.2	17.8	8.2	6.4	8.2	7.2	6.1	5.7	13.3
H	4.1	4.1	7.1	5.6	7.4	1.2	10.1	4.3		5.5	13.5	12.4	10.6	12.4	11.4	10.3	10	17.5
I	9.6	9.5	9	10.3	9.4	4.6	5.6	1.2	5.5		18.6	6.8	6.7	6.8	5.8	4.7	3.3	11.9
J	9.4	11.9	10.2	8.9	12.1	14.4	23.6	17.8	13.5	18.6		26	24.1	25.9	25	23.9	23.5	34.7
K	16.5	16.4	15.9	17.2	16.3	11.8	1.5	8.2	12.4	6.8	26		4.7	3.2	1.4	3.2	3.2	9.3
L	14.7	14.6	14.1	15.4	14.5	10	4.9	6.4	10.6	6.7	24.1	4.7		2.8	5.4	6.3	5	13.5
M	16.5	16.4	15.9	17.2	16.3	11.8	3.3	8.2	12.4	6.8	25.9	3.2	2.8		3.6	4.5	4	11.6
N	15.6	15.5	14.9	16.2	15.3	10.8	0.5	7.2	11.4	5.8	25	1.4	5.4	3.6		3	2.8	8.9
O	14.1	14.3	13.8	15.1	14.2	9.7	2.2	6.1	10.3	4.7	23.9	3.2	6.3	4.5	3		2.2	7.5
P	14.1	14	13.5	14.7	13.8	9.4	2.2	5.7	10	3.3	23.5	3.2	5	4	2.8	2.2		9.2
Q	21.6	21.5	21	22.3	21.4	16.9	8.6	13.3	17.5	11.9	34.7	9.3	13.5	11.6	8.9	7.5	9.2	

IV: *Thursday*

	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
Brewery – Z		6.7	15.7	20	14.5	4.6	16.3	13.6	13.6	17.5	11.3	14.4	16	3	20.7	16.6	40.2	9.1	32.1
A	6.7		12.8	17.7	19.9	9.1	14	11.3	11.2	15.2	11.5	12	13.7	6.4	17.6	14.3	37.2	6.8	29
B	15.7	12.8		14	36.5	20.5	11.1	4.2	7.9	8.6	10.6	9.2	6.9	16.2	6	8.2	25.6	6.8	17.4
C	20	17.7	14		31.3	22	5.2	9.3	6.5	7.6	8.9	6.4	7.5	19.3	12	7.1	12	11	9.1
D	14.5	19.9	36.5	31.3		11.2	28.4	26.9	30.4	32.9	28.3	27.8	31.4	12.9	37.4	32	39	22.4	34.9
E	4.6	9.1	20.5	22	11.2		19.4	16.7	16.7	20.6	14.4	17.5	19.1	2.7	23.8	19.7	39.3	12.2	35.2
F	16.3	14	11.1	5.2	28.4	19.4		8.6	5.6	7.3	6.3	3.7	7.2	16.4	12.8	7	17.3	8.1	12.1
G	13.6	11.3	4.2	9.3	26.9	16.7	8.6		4.2	4.4	6.1	5.6	2.7	15.4	7.3	4	21.2	4.4	16.9
H	13.6	11.2	7.9	6.5	30.4	16.7	5.6	4.2		3.4	3.9	3.4	2.4	12.8	8.4	2.9	17.8	4.5	14.4
I	17.5	15.2	8.6	7.6	32.9	20.6	7.3	4.4	3.4		6	5.5	2.3	16.5	7.4	0.8	18.6	7.1	9.8
J	11.3	11.5	10.6	8.9	28.3	14.4	6.3	6.1	3.9	6		2.5	4.9	13.1	10.9	5.5	24.3	5	16.9
K	14.4	12	9.2	6.4	27.8	17.5	3.7	5.6	3.4	5.5	2.5		4.6	13.4	10.6	5.2	19.9	5.1	14.7
L	16	13.7	6.9	7.5	31.4	19.1	7.2	2.7	2.4	2.3	4.9	4.6		14.4	6.8	3.3	21.8	5	12.9
M	3	6.4	16.2	19.3	12.9	2.7	16.4	15.4	12.8	16.5	13.1	13.4	14.4		21.1	17.1	36.6	9.6	32.5
N	20.7	17.6	6	12	37.4	23.8	12.8	7.3	8.4	7.4	10.9	10.6	6.8	21.1		7.4	20.8	13.9	11.5
O	16.6	14.3	8.2	7.1	32	19.7	7	4	2.9	0.8	5.5	5.2	3.3	17.1	7.4		19	6.5	10.2
P	40.2	37.2	25.6	12	39	39.3	17.3	21.2	17.8	18.6	24.3	19.9	21.8	36.6	20.8	19		22.5	11.6
Q	9.1	6.8	6.8	11	22.4	12.2	8.1	4.4	4.5	7.1	5	5.1	5	9.6	13.9	6.5	22.5		19
R	32.1	29	17.4	9.1	34.9	35.2	12.1	16.9	14.4	9.8	16.9	14.7	12.9	32.5	11.5	10.2	11.6	19	

V: *Friday*

	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
Brewery – Z		5.7	16.4	19.3	9.6	8.6	15.7	8.4	9.9	12.5	14.8	14	20.7	5.5	15.9	17.7	15.6	9.6	14.8
A	5.7		16.1	17.7	9.4	8.4	15.5	8.2	9.6	12.2	14.5	13.7	20.4	8.1	15.6	17.4	15.3	9.4	14.5
B	16.4	16.1		7.1	6.6	7.3	1	8.2	7.4	4.2	2.5	3.3	7.9	18.3	1.8	7.5	3.2	5.9	2.5
C	19.3	17.7	7.1		8.1	8.7	6.5	11.6	9.5	8.8	7.5	5.1	3	22.3	5.7	12.4	8.2	8.1	7.5
D	9.6	9.4	6.6	8.1		0.7	5.9	1.6	0.8	2.8	4.6	6	12.6	12.3	6.2	8	5.8	0.1	4.6
E	8.6	8.4	7.3	8.7	0.7		7.7	1	1	4.5	6.8	6	12.6	11.6	7.8	9.6	7.5	0.8	6.8
F	15.7	15.5	1	6.5	5.9	7.7		7.5	6.6	3.4	1.4	2.7	7.4	17.5	1.2	6.3	2.2	5.8	1.4
G	8.4	8.2	8.2	11.6	1.6	1	7.5		1.6	4.5	6.3	6	12.7	11.3	7.9	9.7	7.6	1.4	6.1
H	9.9	9.6	7.4	9.5	0.8	1	6.6	1.6		2.9	5.2	6.2	12.9	12.5	6.3	8.1	5.9	0.2	4.7
I	12.5	12.2	4.2	8.8	2.8	4.5	3.4	4.5	2.9		2.5	5.1	9.7	14.4	3.6	6.2	3.3	2.8	2.5
J	14.8	14.5	2.5	7.5	4.6	6.8	1.4	6.3	5.2	2.5		3.6	8.3	16.7	2.2	5	0.8	4.5	0.1
K	14	13.7	3.3	5.1	6	6	2.7	6	6.2	5.1	3.6		7.3	16.7	2.7	8.1	5.2	3.8	4.5
L	20.7	20.4	7.9	3	12.6	12.6	7.4	12.7	12.9	9.7	8.3	7.3		25.9	6.8	13.3	9.1	9.5	8.4
M	5.5	8.1	18.3	22.3	12.3	11.6	17.5	11.3	12.5	14.4	16.7	16.7	25.9		18.9	20.7	18.6	12.7	17.9
N	15.9	15.6	1.8	5.7	6.2	7.8	1.2	7.9	6.3	3.6	2.2	2.7	6.8	18.9		7.5	3.3	5.5	2.6
O	17.7	17.4	7.5	12.4	8	9.6	6.3	9.7	8.1	6.2	5	8.1	13.3	20.7	7.5		5.4	7.9	5.7
P	15.6	15.3	3.2	8.2	5.8	7.5	2.2	7.6	5.9	3.3	0.8	5.2	9.1	18.6	3.3	5.4		5.6	0.7
Q	9.6	9.4	5.9	8.1	0.1	0.8	5.8	1.4	0.2	2.8	4.5	3.8	9.5	12.7	5.5	7.9	5.6		4.6
R	14.8	14.5	2.5	7.5	4.6	6.8	1.4	6.1	4.7	2.5	0.1	4.5	8.4	17.9	2.6	5.7	0.7	4.6	

SUMMARY

The problem of the delivery optimization of specific material can in reality be encountered very often. The delivery is usually realized by a circular or round trip which, in comparison with the implementation of each route from the supplier to the consumer, saves expenses for individual gateways from the same supplier and/or trips to one consumer. There exist many kinds of this task. In general, they are referred to as vehicle routing problems (VRP). Practically all of them belong to the NP-hard problems, for which there is no efficient algorithm finding their theoretical optimum. Thus, the only way to obtain some solution efficiently or in a reasonably short time is to use some of

the heuristics (approximation methods), which give only “good” or “close to optimal” solution, not exactly optimum.

In practice, however, companies seldom pay enough attention to dealing with VRPs, especially if transportation is not their principal work load and if a transportation task of a medium size only is concerned. This situation occurs also in a case study of a local brewery situated near to Prague presented in this paper. The aim of this paper is to choose suitable heuristics and propose appropriate methodology for planning the transportation of beer to the customers.

The following approximation methods were selected for examination: the Mayer method, the savings method by Clarke and Wright, Habr frequencies approach and other three types of the combination of two lastly mentioned ones. In case of the Mayer method, single routes were sorted by the Branch and Bound method. During the selected week, the transportation was observed. For each day, the transportation plan was computed by all the methods mentioned above and compared with the routes actually implemented by the brewery. All the methods improved the original transportation plan. In single days, the best solution was found by a different method, but all the methods mostly provided solution of not much different quality. Therefore, we propose to create a macro in VBA in Excel where all these methods are used. As most of these methods are very similar, the application of all these methods did not make programming the macro more complicated than in case of using only one of the method. Moreover, the “manual” computation without using the VBA macro is also relatively easy. Based on the results of the observed week, the brewery can save more than 50,000 CZK (2,000 EUR) per year.

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