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DRIVER'S INFLUENCE ON KINEMATICS OF ARTICULATED BUS

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Abstract

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Driver's influence on kinematics of articulated bus. This paper studies kinematics properties of the particle coach as function of driver's activity. The main goals are the prediction of the trajectory, the computation of the vector of velocity of each wheel as a function of the real velocity vector of the midpoint of the middle axle and the real curvature of the bus trajectory. The computer algebra system Maple was used for all necessary computations. Article is divided into four main parts. In the first part are derived integral relations describing trajectory of the middle axes midpoint as a function of the absolute value of the velocity and trajectory curvature. At the second part is computed joint trajectory and differential equation describing relation between joint position and position of the midpoint of the rear – towed axle. At the third part is shown how to integrate such system of integral-differential equation using Runge-Kutta method and how to estimate proper size of the time step. In the final part is shown how to compute curve passing time and graphical results of the numerical solution are presented.

kinematics, trajectory curvature, generalized tractrix, system of differential equations, numerical integration, Maple

Classical problem of kinematics

In the following computations we should use these main variables: $X(t)$, $Y(t)$ – general coordinates of the moving body, later coordinates of the midpoint of the central axle of the articulated bus. $x(t)$, $y(t)$ – coordinates of the midpoint of the towed – rear axle. $\xi(t)$, $\eta(t)$ – coordinates of the joint of the bus. L – constant distance between midpoint of the rear axle and joint. k – constant distance between midpoint of the central axle and joint, see Fig. 1.

The classical problem of kinematics is the computation of the speed $\boldsymbol{V}(t)$ and the acceleration vector $\boldsymbol{A}(t)$ of a body as a function of time when the location of the body is given by the functions $\boldsymbol{P}(t) = [X(t), Y(t)]]$. The next step is the computation of the tangential acceleration $A_t(t)$, which changes the absolute value of the velocity and the normal acceleration $A_n(t)$, which changes the direction of the velocity. And finally, the function of the center of the osculation circle of the trajectory $C(t)$ and its radius $R(t)$ are derived.

These functions can also be found in (BRAND 1947, SPALLEK 1980, OR WEBFYZ).

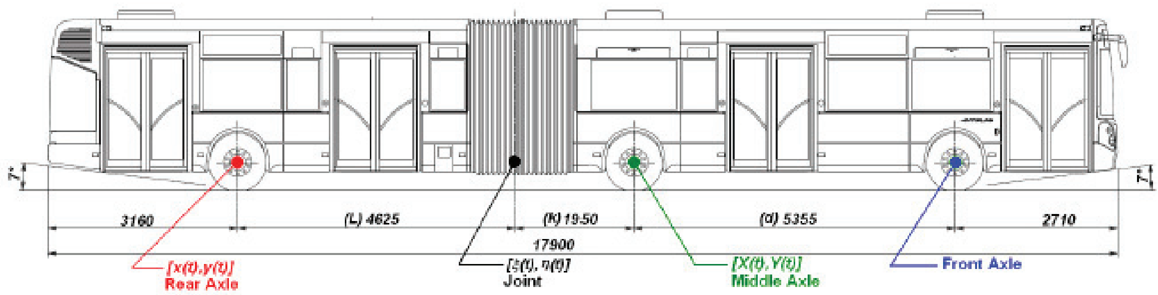
The influence of the driver

The driver controls the bus using the gas and the brake pedal – he controls the absolute value of the velocity of the bus $|\boldsymbol{V}(t)|$. Furthermore – using the steering wheel he controls the radius of the osculating circle $R(t)$, on which the bus is currently moving. For further calculations it is useful to use the inverse value of the radius of the osculation circle – the curvature of trajectory $\kappa(t) = R(t)^{-1}$. By combining these two controls the bus driver keeps the bus moving smoothly on the road.

MATERIALS AND METHODS

Inverse problem

Let us assume that we know the temporal behavior of driver's operations. Thus we know the functions of the speed magnitude $|\boldsymbol{V}(t)| = v(t)$ and curvature $\kappa(t)$.



1: *Main variables and dimensions of the bus Irisbus Citelis 18 m*

Then the problem is to compute the trajectory of the bus and the related kinematics variables. For this we need to solve a non-linear system of two ordinary differential equations of second and first order, they are solved in (BARTOŇ, 2011, or KRUMPHOLC, 2012).

$$\sqrt{\dot X^2+\dot Y^2} = v(t)\,,\quad \frac{\ddot Y\,\dot X - \ddot X\,\dot Y}{\left(\dot X^2+\dot Y^2\right)^{3/2}} = \kappa(t)\,. \tag{1}$$

After some algebraic manipulations the equations (1) are transformed to an explicit system of two differential equations of order two:

$$\ddot X = \frac{-\dot Y\,\kappa(t)\,v(t)^2 - \dfrac{d\,v(t)}{dt}\,\dot X}{v(t)}\,,\quad \ddot Y = \frac{-\dot X\,\kappa(t)\,v(t)^2 + \dfrac{d\,v(t)}{dt}\,\dot Y}{v(t)}\,. \tag{2}$$

Given an initial velocity $v_0 = |\boldsymbol{V}(0)|$ and its initial direction defined by the angle ϕ_0 and the initial position of the bus $[X_0, Y_0]$, the solution of (2) can be found to be, see (BARTOŇ, 2011; KRUMPHOLC, 2012).

$$X = \int_0^t v(\tau)\cos(f)\,d\tau + X_0\,,\quad Y = \int_0^t v(\tau)\sin(f)\,d\tau + Y_0\,, \tag{3}$$

where

$$f = \varphi_0 + \int_0^\tau v(\tau)\,\kappa(\tau)\,d\tau\,.$$

This is an analytic solution, however, even for simple functions and it will not be possible to compute explicit expressions for the integrals. A considerable advantage of this result is that it allows to numerically integrate the position for any given time. We have not to be concerned with accumulation of rounding errors as e.g. by integrating the system (2) with some numerical methods, like Runge-Kutta, see (RALSTON 1978, REKTORYS 2000).

Generalized tractrix as model of the trajectory of the rear axle

The distance between joint of the articulated bus and the midpoint of the middle axle is k. Trajectory of the joint is given by $[\xi(t), \eta(t)]$ and can be expressed as, see Fig. 2.

$$[\xi,\eta] = [X,Y] - k\frac{\left[\dot X,\dot Y\right]}{v(t)}\,. \tag{4}$$

Trajectory of the midpoint of the middle axle is given by $[X, Y]$ and the trajectory of the centre of the rear axle given by $[x, y]$ – the towed axle – is to be computed, see Fig. 1 and Fig. 2. The distance between the joint and midpoint of the rear axle has a constant distanceand the velocity vector of the center of the rear axle has to pass the joint, see Fig 2. These conditions may be expressed as:

$$\frac{\dot\xi}{\dot\eta} = \frac{\xi - x}{\eta - y}\ \text{and}\ (\xi - x)^2 + (\eta - y)^2 = L^2\,. \tag{5}$$

From Equations (5) we obtain the system of differential equations $[\dot x, \dot y]$, see (GANDER, 2004).

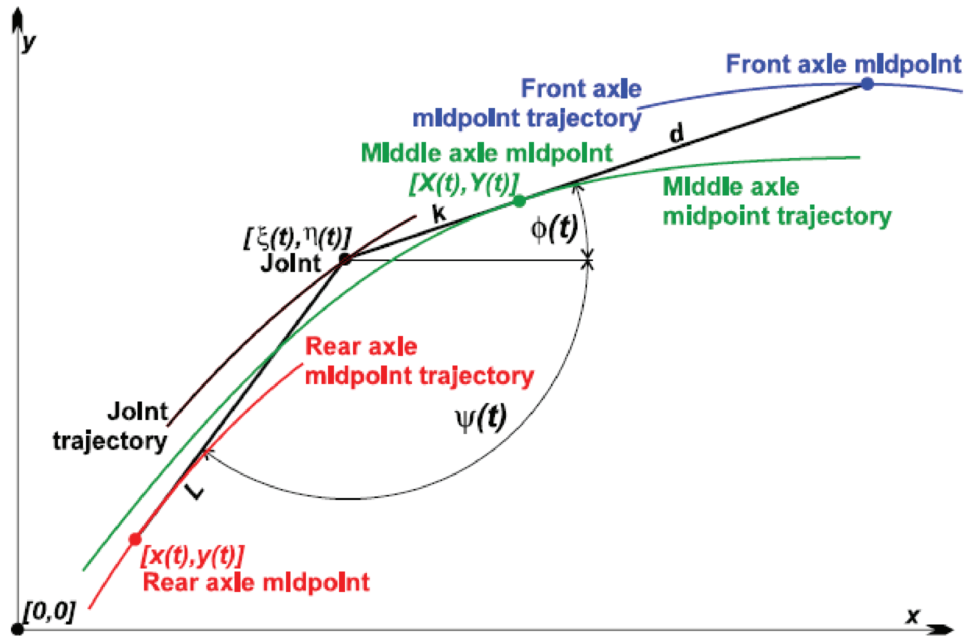
$$\dot x = \frac{\Delta x\left(\Delta x\dot\xi + \Delta y\dot\eta\right)}{L^2}\,,\quad \dot y = \frac{\Delta y\left(\Delta y\dot\xi + \Delta y\dot\eta\right)}{L^2}\,,$$

where (6)

$$\Delta x = x - \xi,$$
$$\Delta y = y - \eta.$$

It is a system of two non-linear differential equations of first order, which for simple functions X and Y, remember equation (4), is relatively easy to solve.

But if we introduce for X and Y the expressions of Equations (3), we get a very complex system of differential equations, for which it is first necessary to solve for X and Y by the numerical integration. This combination of numerical integration and solving of differential equations is too complex for the computer algebra system MAPLE. It is not possible to use successfully direct numerical solution of equations using the MAPLE command



2: *Deriving the motive equation of the rear axle*

dsolve together with the parameter **numeric**, see (MAPLE 2009).

Numerical integration of the equation of motion

It is possible to solve the system (3) together with equations (6) numerically using Runge-Kutta's method, see (RALSTON 1978, REKTORYS 2000). We implemented this in MAPLE as procedure **RK45**, see (7).

$$\begin{aligned}
&\mathbf{RK45} := \mathbf{proc}(t,x,y,\Delta t)\ \ \mathbf{local}\ K_1,K_2,K_3,K_4,L_1,L_2,L_3,L_4;\\
&\quad K_1 := \mathbf{evalf}(\dot{x}(t,x,y));\ L_1 := \mathbf{evalf}(\dot{y}(t,x,y));\\
&\quad K_2 := \mathbf{evalf}(\dot{x}(t+\frac{\Delta t}{2}, x+\frac{\Delta t}{2}K_1, y+\frac{\Delta t}{2}L_1));\\
&\quad L_2 := \mathbf{evalf}(\dot{y}(t+\frac{\Delta t}{2}, x+\frac{\Delta t}{2}K_1, y+\frac{\Delta t}{2}L_1));\\
&\quad K_3 := \mathbf{evalf}(\dot{x}(t+\frac{\Delta t}{2}, x+\frac{\Delta t}{2}K_2, y+\frac{\Delta t}{2}L_2));\\
&\quad L_3 := \mathbf{evalf}(\dot{y}(t+\frac{\Delta t}{2}, x+\frac{\Delta t}{2}K_2, y+\frac{\Delta t}{2}L_2));\\
&\quad K_4 := \mathbf{evalf}(\dot{x}(t+\Delta t, x+\Delta t\, K_3, y+\Delta t\, L_3));\\
&\quad L_4 := \mathbf{evalf}(\dot{y}(t+\Delta t, x+\Delta t\, K_3, y+\Delta t\, L_3));\\
&\quad [t+\Delta t,\ x+\frac{\Delta t}{6}(K_1+2K_2+2K_3+K_4),\\
&\qquad y+\frac{\Delta t}{6}(L_1+2L_2+2L_3+L_4)][]\\
&\mathbf{end\ proc}
\end{aligned} \tag{7}$$

This procedure determines the position and velocity of the towed axle's centre at time $t + \Delta t$. The next procedure, named **STEP**, see (8), defines the magnitude of time step Δt using a step size control.

$$\begin{aligned}
&\mathbf{STEP} := \mathbf{proc}(U)\ \mathbf{local}\ l, R2;\\
&\quad \mathbf{global}\ R1,\Lambda,\Delta t,t;\\
&\quad l := U[];\ R2 := [\mathbf{RK45}(\mathbf{RK45}(l,\frac{\Delta t}{2}),\frac{\Delta t}{2})];\\
&\quad R1 := [\mathbf{RK45}(l,\Delta t)];\\
&\quad \mathbf{if\ sqrt(add}(u^2,\ u = R1-R2)) \le 10^{-6}\\
&\qquad \mathbf{then}\ \Lambda := [\Lambda[], R1];\ t := t+\Delta t;\\
&\qquad \mathbf{else}\ \Delta t := \frac{\Delta t}{2}\ \mathbf{end\ if}\\
&\mathbf{end\ proc}
\end{aligned} \tag{8}$$

For the first iteration a random time step magnitude is chosen, e.g. $\Delta t = 1s$ and the position for this time is calculated. Time t and coordinates x, y are saved in the vector $R1$. Similarly the position is calculated in the same procedure, but in two steps with a half time step size $\Delta t/2$ and saved as a vector $R2$. If the difference between these vectors is smaller than the required accuracy, $|R1 - R2| \le 10^{-6}$, we add the resulting position, saved in the vector $R1$ to vector Λ. Otherwise we reduce the size of time step by half and repeat the process. At the end of the iteration procedure the vector will contain vectors – ordered triplets containing the time and the towed axle's position coordinates of the each iteration step.

Practical application

Let's take as example a passing of a rectangular turn when the bus is breaking. For this case we consider

$$v(t) = V_0 - a\,t,\ \kappa(t) = \frac{4t\,(T_f - t)}{T_f^2\,\rho}, \qquad (9)$$

where
V_0......the initial velocity,
a ....deceleration,
T_f......the period of turn passing and
ρ ....least radius of a passed turn.

If we choose the direction in time $t = 0$ parallel to x axis, therefore $\phi_0 = 0$, the turn will be finished at the moment, when the vector of immediate velocity $[\dot{X}, \dot{Y}]$ will be parallel to y. Therefore it is stated that $\dot{X} = 0$. From this condition it is obvious, that because of Equation (3) we have

$$T_f = \frac{2V_0 - \sqrt{4V_0^2 - 6a\pi\rho}}{2a}. \qquad (10)$$

Details can be found in (KRUMPHOLC, 2011A; KRUMPHOLC, 2011B).

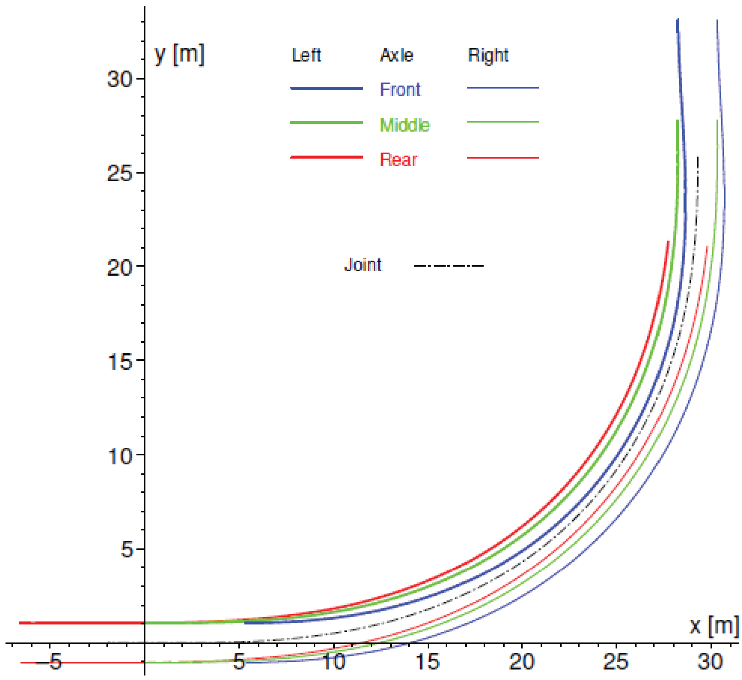
RESULTS

As particular values we take $V_0 = 10\ ms^{-1}$, $a = 0.5\ ms^{-2}$, $\rho = 20\ m$, $x_0 = 0\ m$, $y_0 = 0\ m$, $k = 1.950\ m$ and $L = 4.625\ m$, see Fig (1) and (IRISBUS Citelis, IRISBUS Citelis 18 m). For these values the time necessary to pass the turn is $T_f = 5.45681\ s$. The initial time is $t_0 = 0\ s$ and for the initial time step we choose $\Delta t = T_f$. Now we create the list, its first element will be $[t, -L, 0]$, then $\Lambda := [[0, -4, 0]]$. Procedure **STEP**determines the first step size of the time step as $\Delta t = T_f/128 = 0.04263\ s$ and then executes 128 integration steps. For specific integration times it is possible to compute using Equations (3) the position of middle axle's centre point. Due to Fig. 2 and the following relationship (11) it is hence possible to calculate the position of front, middle and rear – towed axle.

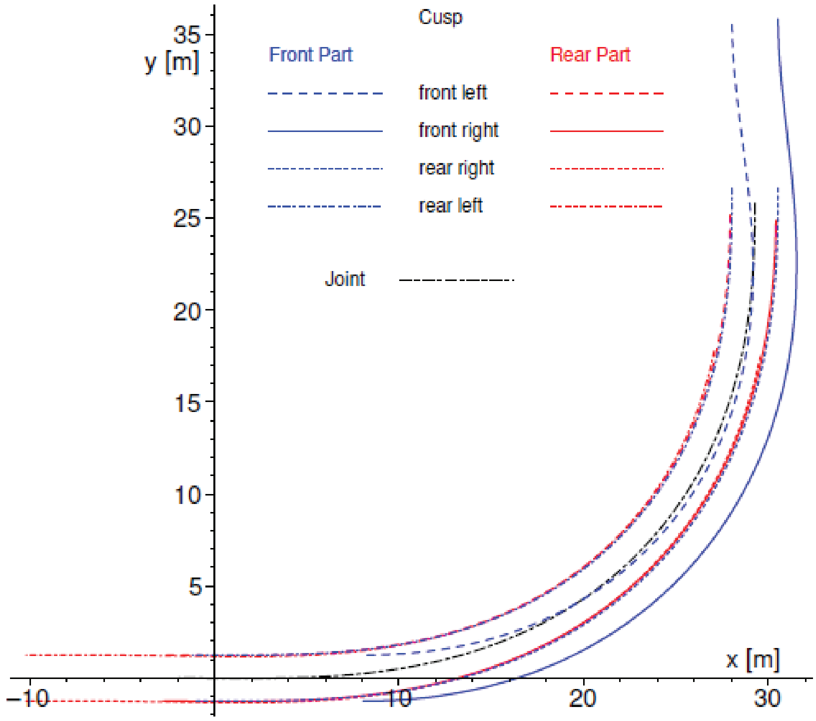
$$Wheel = [\xi, \eta] + D[\cos(\alpha), \sin(\alpha)] + r[-\sin(\alpha), \cos(\alpha)] \qquad (11)$$

for D we can take k or $k + d$ - the distance between joint and center point of middle or front axle, or L – the distance between joint and rear axle's center point. For a we can take ψ – the directional vector pointing from the joint to the middle axle's midpoint. This is the directional vector of the velocity $[\dot{X}, \dot{Y}]$ or ϕ – the directional vector pointing from the joint of the middle point of the rear axle, r – wheel spacing of single axles. Angular sizes ϕ and ψ could be easily solved using the vector calculation. The result of the calculation could therefore be a graph on Fig. 3, depicting the trajectories of single wheels, or a graph on Fig. 4, which represents the trajectory of the cusps of the bus body.

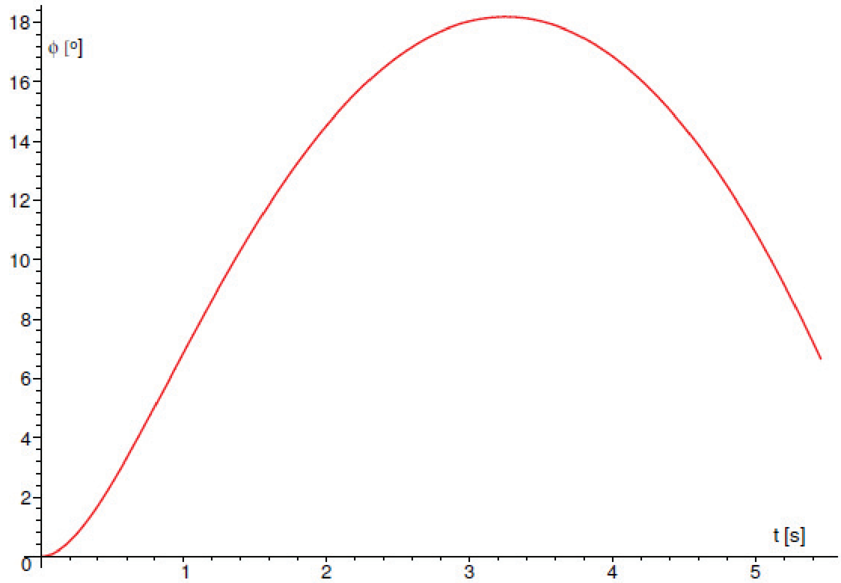
The angle of cranking of the bus joint is shown on Fig. 5. The whole bus at five positions in the curve is shown on Fig 6. As is shown on this figure, difference between trajectory of the cusp and of the wheel is greater as one meter.



3: *Trajectory of separate wheels of the bus*



4: *Trajectory of the bus body*

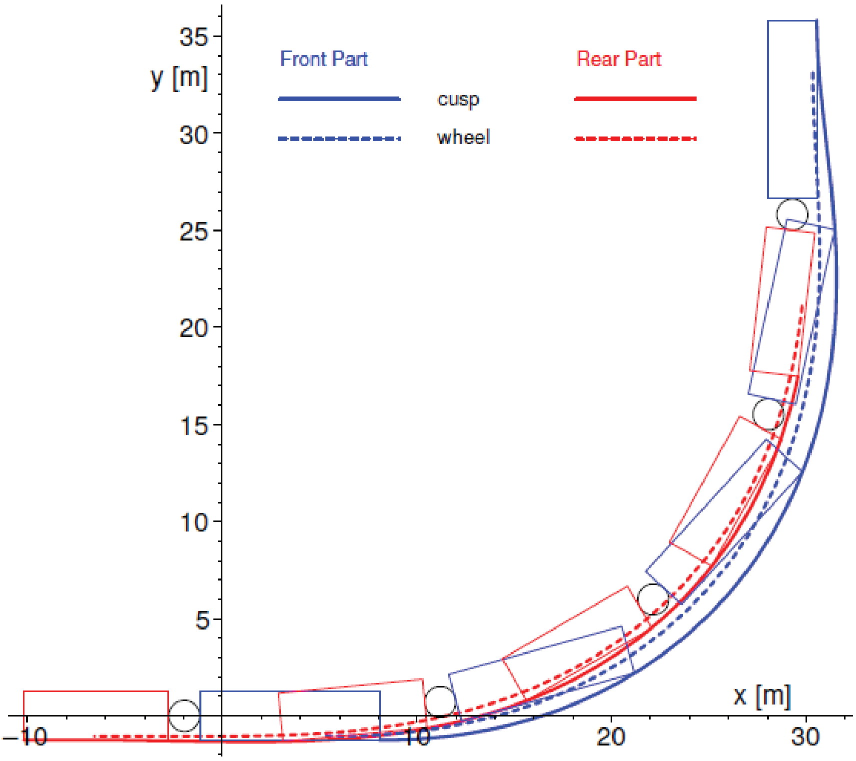


5: *Angle of cranking the joint of the bus*

CONCLUSION AND DISCUSSION

Procedures presented in this article could be also used for the inverse problem. From the moment of the adhesion loss to breakaway it is possible to experimentally find such a velocity and trajectory curvature functions that caused the skid. Therefore it is possible from the trajectory – a braking track – to estimate the driver's actions that preceded this event.

From the knowledge of acceleration inside the bus it is possible to perform the calculations of general force, affecting the whole bus as well as individual passengers. Knowledge of this general force is an important factor affecting the stability of the bus. The force affecting the single passenger is a limiting factor for their safety. The method mentioned above allows us to simulate the driver's behavior and the impact on safety of passengers due to their position inside the bus.



6: *Bus positions in a curve*

SUMMARY

We have developed a method which allows for any absolute value of the bus velocity function given by the $v(t)$ and trajectory curvature function $k(t)$ to compute all important kinematics variables of the articulated bus. This concerns not only the wheels but can be applied for any arbitrary point inside the bus. Just take for that the appropriate dimensions of the articulated bus corresponding to its technical descriptions. This approach can be used to study behavior of an arbitrary truck or so called "road train". Furthermore, it is possible to determine the acceleration of any point, including the points which correspond to points of contact between the wheels and the road. This knowledge of acceleration could be used for the determination of the adhesion threshold limits. Knowledge of the acceleration of the arbitrary point of the articulated bus can be used for computations of the forces and their torque moments acting to the joint of the bus or truck. Torque moments are direct causes of the skid, so presented algorithms may be used for computer modeling of the behavior of the bus before and during skid.

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