ACTA UNIVERSITATIS AGRICULTURAE ET SILVICULTURAE MENDELIANAE BRUNENSIS

Volume LX 50 Number 2, 2012

PHYSICAL AND EXPENDITURE FORMULATIONS OF DEMAND FUNCTIONS AND EVALUATIONS OF PRICE DEMAND ELASTICITY

P. Syrovátka

Received: November 30, 2011

Abstract

SYROVÁTKA, P: *Physical and expenditure formulations of demand functions and evaluations of price demand elasticity.* Acta univ. agric. et silvic. Mendel. Brun., 2012, LX, No. 2, pp. 379–382

Studies of the demand relations on the consumer markets bring much useful information. The concept of the elasticity coefficients is frequently used for the quantitative analysis of the demand sensitivity. Formulation of the investigated demand functions is very important for the evaluation of the demand elasticity. Within net consumer demand (consumer purchase), it is possible to differentiate the physical and expenditure forms of the demand functions. The paper is focused on the theoretical and methodological backgrounds of the evaluation of price-demand elasticity under the physical and expenditure definitions of the demand relationships. In this paper, the relationship between the coefficient of the price elasticity of demand in the physical form and the coefficient of the price elasticity of demand in the expenditure form is determined and studied. The derived formula is tested using the USDA database.

consumer demand, physical form of demand, expenditure form of demand, price-demand elasticity, price-expenditure elasticity

Quantitative analyses of the demand functions within the consumer markets give the irreplaceable input information for the studies of market structures and/or partial market equilibriums, see Koutsoyiannis, A. (1979) or Deaton, A. (1986). These demand analyses are also important for the researches of the market interactions especially within the product chains, see Cramer, G. L., Jensen, C. W. (1994). Determinations of the demand functions of the processors and producers are linked to the final consumer demand, see Bečvářová, V. (2008). With respect to the mentioned connections, the quantitative analyses of the own price sensitivity of the consumer demands are primarily important. The evaluations of the own price sensitivity of the consumer demand are based on the calculations of the elasticity coefficients. The own price-demand elasticity may be investigated under the physical and/or expenditure definitions of related demand functions, see Deaton, A. (1986) or Nicholson, W. (1992).

Preliminary assumptions

Let us assume that the market price only affects the level of consumer demand. The demand level may be measured in both the physical units (kilograms, litres, peaces, metres etc.) and monetary units (expenditure for the good). Thus, the physical form of the price-demand function is declared as follows:

$$q = f(p). \tag{1}$$

In the demand function (1), the variable represents the volume/size/quantity of the consumer demand for the given goods and independent variable is the market price of this good. Then it is possible to write the expenditure form of the price-demand function as follows:

$$x = f(p). \tag{2}$$

The depended variable measures the expenditure for the observed good and the monetary level of this expenditure is defined by the product (3):

$$q = p \times q. \quad (3)$$

Within the physical form of the consumer demand function (1), the coefficient of own price elasticity is calculated as follows:

$$\varepsilon = \partial q/\partial p \times p/q\ . \quad (4)$$

In accordance with the following formula:

$$\chi = \partial x/\partial p \times p/x\ , \quad (5)$$

the own price elasticity of the expenditure form of the consumer demand function (2) is determined.

Price elasticity of demand and expenditure – interrelation of related elasticity coefficients

According to the formulas (1) and (3), we can formulate the consumer demand function in the product term:

$$x = p \times q(p). \quad (6)$$

Applying the differentiation rules about product, we get the derivative of the demand function (6) with respect to the market price. The result of the derivative process is:

$$\frac{\partial x}{\partial p} = q(p) + p \times \frac{\partial q(p)}{\partial p}\ , \quad (7)$$

thus a differential equation. If we multiply the second addend on the right side of (7) by the ineffective term: $q(p)/q(p) = 1$, we achieve the equation:

$$\frac{\partial x}{\partial p} = q(p) + p \times q(p) \times \frac{\partial q(p)}{\partial p} \times \frac{p}{q(p)}\ . \quad (8)$$

With regard to the definition of the price elasticity coefficient (4), the equation (8) may be rewritten into following form:

$$\frac{\partial x}{\partial p} = q(p) + q(p) \times \varepsilon\ . \quad (9)$$

or rearranged into the term:

$$\frac{\partial x}{\partial p} = q(p) \times [1 + \varepsilon]\ . \quad (10)$$

Multiplying both sides of the equation (10) by term p/x, we obtain:

$$\frac{\partial x}{\partial p} \times \frac{p}{x} = \frac{p}{x} \times q(p) \times [1 + \varepsilon]\ . \quad (11)$$

The right side of the achieved equation (11) represents the coefficient, i.e. the coefficient of the price elasticity of expenditure (5). In accordance with the product (6): $x = p \times q(p)$, we are able to continue and to simplify the equation (11) as follows:

$$\chi = 1 + \varepsilon. \quad (12)$$

The reduced equation (12) shows clearly the mutual relationship between the coefficient of price-demand elasticity (ε) and the coefficient of price-expenditure elasticity (χ).

RESULTS, FINDINGS AND DISCUSSION

According to the determined interrelationship (12), it is possible to declare the links between the interval of values of the ε coefficient and χ coefficient. A complete overview of the achieved results is depicted in Table I.

Table I shows the elastic negative reactions of the consumer expenditure to price changes within the category of the ordinary goods with the coefficient

I: *Demand responses to price changes*

Physical form of consumer demand		Expenditure form of consumer demand	
Price-demand reactions	**Values of price elasticity coefficient**	**Price-expenditure reactions**	**Values of price elasticity coefficient**
Negative – elastic	$\varepsilon \in (-\infty; -2)$	Negative – elastic	$\chi \in (-\infty; -1)$
	$\varepsilon = -2$	Negative – unit elasticity	$\chi = -1$
	$\varepsilon \in (-2; -1)$	Negative – inelastic	$\chi \in (-1; 0)$
Negative – unit elastic	$\varepsilon = -1$	None	$\chi = 0$
Negative – inelastic	$\varepsilon \in (-1; 0)$	Positive – inelastic	$\chi \in (0; +1)$
None	$\varepsilon = 0$	Positive – unit elastic	$\chi = +1$
Positive – inelastic	$\varepsilon \in (0; +1)$	Positive – elastic	$\chi \in (+1; +2)$
Positive – unit elastic	$\varepsilon = +1$		$\chi = +2$
Positive – elastic	$\varepsilon \in (+1; +\infty)$		$\chi \in (+2; +\infty)$

Source: Author calculations

of own price elasticity lower than –2. Within the category of the ordinary goods, it is also possible to record the inelastic negative responses of the consumer expenditure for ones. In this case, the value of the price elasticity coefficient (ε) is higher than –2 and simultaneously lower than –1. Negative reactions of the expenditure for the ordinary goods achieve just unit elasticity if the coefficient of own price elasticity is equal –2. If the demand for the ordinary goods is just unit elastic to the price changes, then the price elasticity of the consumer expenditure for this good) is zero, i.e. no expenditure response runs. Inelastic positive reactions of the consumer expenditure to price changes are manifested within the category of ordinary goods with the inelastic price-demand responses. In case of good with no price-demand reactions (ε = 0), the expenditure responses to the price changes are positive and unit elastic exact. The elastic positive reactions of the consumer expenditure can always be recorded within the category of goods with positive price-demand reactions. In this respect, the intensity (inelastic, unit elastic or elastic) of the price-demand elasticity in not important.

Walter Nicholson came up with similar findings, but his analysis in this field of the consumer behaviour was based on the alternative differential equation: $\partial x/\partial p \times 1/q(p) = 1 + \varepsilon$. This differential equation did not allow the evaluation of the price elasticity of consumer expenditure. The author appraised only the negative, positive or no reactions of expenditure to the prices changes under the inelastic, elastic or unit elastic consumer demand, see Nicholson, W. (1992).

The derived differential equation (12) also offers the possibility of the determination of the price expenditure elasticity, when the value of the price elasticity of the related consumer demand is known and vice versa. The values of the price-expenditure elasticity may be used for the price-sensitivity analysis of the revenues, because the consumer expenditures are simultaneously the revenues of the sellers. Table II shows the application of (12) for the quantification the price elasticity of the expenditure for the broad consumption groups (χ). For these calculations, the values of the price-demand elasticity (ε) were obtained from the USDA database (2005) – the coefficients of the uncompensated price-demand elasticity for the aggregate consumption categories, Czech Republic.

The achieved results of χ coefficients (see Table II) demonstrate that the growth of the price level raises the expenditure for all observed groups of the consumer goods. These positive responses of the consumer expenditure are inelastic: $0 < \chi < 1$, which is given by the inelastic price reactions of the related demands: $-1 < \varepsilon < 0$. The consumer expenditure for the group of food, beverages and tobacco proved the highest positive responses to the increase of the price level. The price increase of 1% caused the growth of the expenditure for food, beverages and tobacco by 0.48 %. Almost no reactions of the consumer expenditure to the price changes were found for the group of medical and health (0.07%) and the group of recreation (0.03%). In these consumption categories, the price-demand elasticity converges to –1 %, thus given demand functions are approximately unit price elastic to the price changes.

II: *Calculations of price-expenditure elasticity for broad consumption groups*

Consumption group	Price-demand elasticity[a]: ε	Price-expenditure elasticity[b]: χ
Food, beverages and tobacco	–0.514	0.486
Clothing and footwear	–0.723	0.277
Housing	–0.821	0.179
Housing furnishing	–0.783	0.217
Medical and health	–.932	0,068
Transport and communication	–0.866	0,134
Recreation	–0.975	0.025
Education	–0.681	0.319

Source: [a]USDA and [b] author calculations

CONCLUSION

The paper is focused on the theoretical and methodological backgrounds of the evaluation of price-demand elasticity under the physical and expenditure forms of the consumer demand. The mutual relationship between coefficient of the price elasticity of demand in physical units (ε) and coefficient of the price elasticity of demand in monetary units (χ) was defined and studied. The achieved interrelationship for the given coefficients: $\chi = 1 + \varepsilon$ was derived from the differential equation: $\partial x/\partial p = q(p) + p \times \partial q(p)/\partial p$. According to the achieved formula, the links between the intervals of the values of the ε coefficient and χ coefficient were declared. The achieved formula was applied for the calculation of the price elasticity of expenditure for the chosen consumption groups (food, beverages and tobacco; clothing and footwear; housing; housing furnishing; medical and health; transport and communication; recreation; education). Within the calculations of the χ coefficient, the USDA database/Czech Republic item (2005) was used. The coefficients of the uncompensated price elasticity of the Czech demand were received from this database.

The paper was prepared within the Research Plan of the Faculty of Business and Economics, Mendel University of Agriculture and Forestry, MSM 6215648904 Czech Economy in Integration and Globalisation Processes and Development of the Agricultural Sector and Services Sector under the New Conditions of the Integrated Agrarian Market as a Part of Solving the Thematic Specialisation 4 "Development Trends in Agribusiness, Formation of Segmented Markets within Commodity Chains and Food Networks in the Process of Integration and Globalisation and Change of Agrarian Policy".

REFERENCES

BEČVÁŘOVÁ, V., 2008: Issues of Competitiveness of Today's Agriculture. Agricultural Economics. 2008. Vol. 54/9, pp. 399–405. ISSN 0139-570X.

CRAMER, G. L., JENSEN, C. W., 1994: Agricultural Economics and Agribusiness. USA: John Wiley & Sons, p. 534. ISBN 0-471-59552-7.

DEATON, A., 1986: Demand Analysis, Chapter 30, In: Zvi Griliches and Michael D. Intriligator, Editor(s), Handbook of Econometrics, Elsevier, 1986, Vol. 3, pp. 1767-1839, ISSN 1573-4412, ISBN 978-0-44-486187-0.

KOUTSOYIANNIS, A., 1979: Modern Microeconomics. 2nd edition. London, Macmillan, p. 581. ISBN 0-333-25349-3.

NICHOLSON, W., 1992: Microeconomic Theory, Basic Principles and Extensions. 5th edition, USA: Dryden Press, p. 825. ISBN 0-03055043-2.

Address

doc. Ing. Pavel Syrovátka, Ph.D., Ústav ekonomie, Mendelova univerzita v Brně, Zemědělská 1, 613 00 Brno, Česká republika, e-mail: pavels@mendelu.cz