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COMPARISON OF METHODOLOGICAL APPROACHES TO IDENTIFY ECONOMIC ACTIVITY REGULARITIES IN TRANSITION ECONOMY

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Abstract

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Presented paper focuses on consideration and evaluation of methodical approaches to analyze cyclical structure character of economic activity in transition economy. As a starting point, work in time domain is applied, which is followed in frequency domain approach. Both approaches are viewed from methodical as well as application point of view and their advantage and disadvantage are discussed. Consequently, time-frequency domain approach is added and applied on real data. On the basis of obtained results recommendation is formulated. All discussed methodical approaches are also considered from the perspective of capability to evaluate behaving of business cycle in time of global economic crisis before/after year 2008. The empirical part of the paper deals with data of gross domestic product in the Czech Republic in 1996/Q1–2010/Q2.

cyclical structure, time domain, frequency domain, time-frequency domain, business cycle

The issue of modeling the economic cycle is the most important for the stabilizing function of economic policy. Identification of the different phases of cyclical fluctuations in economic activity, including the turning points is crucial not only for implementation of monetary and fiscal policy at the level of national economies, but also in the broader context of economic integration process, especially from the perspective of the theory of optimum currency areas (the OCA). The rate of accuracy of identification of cyclical fluctuations in economic activity, including the turning points increases the monetary and fiscal policy implementation efficiency, especially symmetric and asymmetric shocks identification for optimal monetary and fiscal interactions across the Eurozone. This topic is at the forefront of economists' interests especially in the context of joining euro and monetary policy efficacy.

As Fidrmuc and Korhonen (2006) showed in their meta-analysis, there are several different approaches for measuring the synchronization of economic cycles or their movements which provides different results. Consequently, the key question is based on the economic cycle definition.

The rational explanation for cyclical movements in economic activity may be found in shifting demand and supply curves. This process leads to wage, investments and technology differentials between the sectors in the economy. The economic agents want to achieve high profit at low risk. They are constantly seeking profit which appears. Therefore the agents diversified their investments according to their expectations. This process is persistent in market economies and helps the optimal sources efficiency. Transition economies, which include the Czech Republic, changed from the centrally planned economies to the free market economies. These economies have undergone economic

liberalization, privatization of government-owned enterprises and creation of financial sector. This process is characterized by the changing or creating of institutions, which induced structural changes in economic activity development.

The transition economies are mostly affected by economic decrease, called as transformation depression (Borensztein, Demakas and Ostry, 1992). The most important source was fall in real households' income caused by price liberalization (Neset, 2002). In the case of the Czech Republic, this process starts from the year 1990. During the year 1992 economic activity slightly increased, economic growth revival arrived during the years 1993 and 1996. This trend was supported by low unemployment, stable inflation and government budget surplus. The main causes of the economic growth were investment and demand increase. The overhang in investments was not fully accompanied by savings. This difference was covered by foreign capital investments. The positive interest differential and fixed exchange rate regime motivated portfolio investments, which equalized current account of balance of payment. Subsequently, the banking sector was burdened by bad loans and domestic demand growth exceeds GDP. The inflation pressures were corrected by fiscal and monetary restriction. The years 1997, 1998 and 1999 are usually referred as crisis years, grounded by low economic growth in Germany, Asian crisis and massive decrease of capital account. The external imbalance was accompanied by interest rate increase which led to introduction of managed floating exchange rate regime in the year 1997 (Žídek, 2006).

The occurrences mentioned above are typical structural changes in economy which has temporary effect in the economic activity and cannot be included into the business cycle movements. Similarly, the years 2008–2011 are affected by financial crisis. For that reason, it is very difficult to identify complete business cycles in the Czech Republic. However, *for well over a century, business cycles have run an unceasing round. They have persisted through vast economic and social changes; they have withstood countless experiments in industry, agriculture, banking, industrial relations, and public policy; they have confounded forecasters without number, belied repeated prophecies of a 'new era of prosperity' and outlived repeated forebodings of 'chronic depression* (Burns, 1947).

From the methodological point of view, it is important to answer the question, if transition economy exhibit fluctuations which correspond with the definition of Burns and Mitchell (1946). The key problem of business cycle identification in transition economy is, whether a cycle of fixed periodicity is appropriate.

Analysis of time series taking place in the time domain is unable to affect the frequency characteristics and frequency of time series. On the contrary, frequency domain analysis identifies periodical components in economic activity movements which share many elements that are important for the analysis. It provides a deeper insight into the structure and the cyclical behavior of time series in different time scales, as well as the development of time series decomposition in terms of periodic contribution.

To understand clearly to movements in economic activity is dynamic perspective necessary. The authors not apply only frequency domain analysis but also time-frequency approach which focuses on identification of waves at different frequency and time periods.

The dynamic approach is frequently applied to analyze business cycles in transition economies. Jagrič and Ovin (2004) applied multivariate wavelet analysis of business cycle in Slovenia which undergone the transformation depression during the years 1991 to 1993 (production declined rapidly, by 9.3 per cent in 1991 and 6.0 per cent in 1992). Subsequently, Slovenia reached debt crisis in the year 1995.

As well as the Czech Republic, Slovenia is sensitive to changes in export growth. Very important question is, whether only internal factors are responsible for the cyclical behavior of economic activity. Identification of frequencies and duration of business cycles helps us to understand the business cycle causes which are common in open-market economies.

The business cycle is typically defined in terms of period or frequency of adjustments. The spectral properties calculated by Fourier transformation (Hamilton, 1994; Ján, 1997) highlight a number of important differences in the cyclical properties of various measurements of the economic cycle. This approach is able to provide larger amounts of information, such as finding the frequency and length of cycles, which can be further analyzed in terms of linkage and synchronization of cyclical movements as a result of increased international trade, openness of financial markets, and global capital flows.

Expanding the work from time to frequency domain can be found in the works of Guy and St-Amant (1997), Baxter-King (1999) or Iacobucci (2003). In both works the authors focus attention on the band pass filter, high pass or low pass filters respectively. Frequency domain is also used for optimization of parameters when behavior of the spectrum or the ideal filter approximation is investigated. To increase the use of frequency analysis can be seen in the last decade. Many works using the techniques of frequency analysis was precisely in connection with the European Integration process, the countries examined its transitive economies in the euro area, the mutual alignment of the various aspects of development and the domestic economy. Among contemporary works can be included Woźniak and Paczińskieho (2007) calculated the spectrum using time-series autoregressive process through appropriate final order or work of Rua (2010) measuring co-

movement between chosen cuntries using time-frequency domain representation. Also work of Halleth and Richter (2004), who briefly considered the convergence of euro-area selected for estimating the spectrum of nonstationary processes.

The aim of this paper is to recommend the methodical approach for cyclical structure analysis of growth business cycle in transition type of economy. As dominated work of business cycle analysis is frequently in the time domain, this paper starts with this methodological approach. Thus, it means calculating peak to peak length of the cycle after dating obtained growth business cycle via band-pass filtering. Consequently, analysis continues in frequency domain perspective, where cyclical behaving of business cycle is described by spectrum estimation. In both cases, time and frequency domain, advantages and disadvantages is discussed. As the last methodical approach is taken analysis in time-frequency domain, where behaving of business cycle is described not only in the time, but also in the frequency domain as well.

MATERIAL AND METHODS

Due to the aim of the paper three general steps of methodical approach is considered. The first general step is analysis in time domain. The main task is to identify growth business cycle via chosen filtering methods. This step is basic one also for the next general two steps (frequency and time-frequency domain). In addition to the first general step also dating of obtained growth business cycle and calculation of the identified cycle's length will be added. The second general step is focused on analysis in frequency domain. That is, identification of cycle's length by estimation of the spectra via method described below and testing of significance of identified cycle's length. The last one, the third general step, will consist from analysis in time-frequency domain via wavelet transform.

Let's consider standard additive decomposition of the time series Y_t,

$$Y_t = g_t + c_t + s_t + \varepsilon_t, t = 1,...,n, \tag{1}$$

where g_t denotes long-term trend, c_t is the cyclical component, s_t seasonal component (if the data are not adjusted for seasonality) and ε_t is the irregular component. If the decomposition of time series analysis is applied on seasonally adjusted values, then the seasonal component of the equation above can be removed. Commonly the interest of analytic work is concentrated on estimation and description of the trend function. Thus, the cyclical component after simplification in denotation in relation (1) can be obtained using detrending or filtering of input values as a residue, i.e. $c_t = g_t - Y_t, t = 1,..., n$.

In this paper obtaining the cyclical component using Christiano-Fitzgerald filter (Christiano and Fitzgerald, 1999) and Baxter-King filter (Baxter and King, 1999) with frequency band stated according Burnsch and Mitchell (1946). That is, between 6 ad 32 quarters.

Identification of turning points of growth business cycles in the time domain have been performed using the Bry-Boschan algorithm (Bry and Boschan, 1971). The idea of this alghoritm is at first to identify major cyclical movements, establish surrounding of the highest and the lowest cyclical movements. In the second step analysis of these movements in this surrounding is applied leading to the specification the moment of turning point.[1] Based on identifined moments of trough and peak in time domain peak-to-peak cycle length in the sense of time (number of quarters) between two peaks is calculated.

In the frequency domain the cycle length has been identified on the basis of sample spectra estimate. It can be done in several ways. In the case of nonparametric methods periodogram is a basic one, in the case of parametric methods can be spectrum estimated through autoregressive process (Hamilton, 1994). The basic premise is that the input time series is weakly stationary.

Spectrum of time series Y_t can be expressed by Fourier sum (Hamilton, 1994)

$$S_Y(\omega) = \frac{1}{2\pi} \sum_{j=-\infty}^{+\infty} \gamma_j e^{-i\omega j}, \tag{2}$$

where

$$\gamma_j = \mathrm{cov}\left(Y_t, Y_{t+j}\right) = E(Y_t - \mu_t)(Y_{t+j} - \mu_{t+j})$$

is autocovariance between Y_t a Y_{t+j}, $i = \sqrt{-1}$. For the angular frequency ω in radians holds ω = 2π/n, where *n* is the sample size.

Parametric spectrum estimation using the autoregressive AR model can be calculated as (Proakis *et al.*, 2002),

$$\hat{S}_Y(\omega) = \frac{\sigma_w^2}{\left|1 - \sum_{i=1}^{p} a_i e^{-j\omega in}\right|^2}, \tag{3}$$

where a_i are coefficients of the AR model of order *p*, *n* is the range of the data set (length of input time series). To estimate the parameters of the model *AR*(*p*) with optimum *p* value procedure of Yule-Walker method (Proakis *et al.*, 2002) can be used. To optimize the lag in order autoregressive process information criterion can be used.

1 From advanced methods Markov-Switching model (Harding and Pagan, 2002) or dating using nonparametric kernel estimation (Poměnková, 2010a) can be suggested.

Nonparametric method for estimating the spectrum can be realized by periodogram. The general result known as the spectral representation theorem states that any stationary process Y_t can be expressed in the form

$$Y_t = \mu + \sum_{j=1}^{M} \alpha_j \cdot \cos\left\{\omega_j (t-1)\right\} + \delta_j \cdot \sin\left\{\omega_j (t-1)\right\}, \quad (4)$$

where α_i, δ_i are random variables with zero mean value, i.e. $E(Y_t) = 1$ for all t. Estimates of the parameters $\hat{\alpha}_i$, $\hat{\delta}_i$ can be obtained using the ordinary least squares (OLS) method. Thus, for the stationary process Y_t with absolutely summable sequences of autocovariances for arbitrary ω we can construct value of sample periodogram in frequency according to the formula

$$\hat{S}_Y(\omega_j) = \frac{1}{4\pi}\left(\hat{\alpha}_j^2 + \hat{\delta}_j^2\right), j = 1, 2, \ldots, M. \quad (5)$$

Sample spectrum $S_Y(\omega)$ indicates the proportion of sample variance Y, which can be attributed to the cycle of frequency ω (Hamilton, 1994).

Wavelet transform allows time-frequency (more precisely time-scale) approach for time series analysis. In this transform, any basis function (wavelet) is defined on finite time interval. Then, the properties that are described by certain value of spectrum can be related to the mentioned time interval. Note that the classical frequency transform does not allow that. When periodogram is calculated, Fourier transform with harmonic basis functions which are nonzero on the whole time interval is used. Then, estimate of the spectrum in each point is influenced by the whole time series. On the other hand, the wavelet basis functions cover the time interval of the series per parts (wavelet basis function is time shifted and scaled), so that complete time-frequency information can be expressed.

Regarding the ability of temporal localization of events in the spectrum, then the wavelet transform to a certain extent, be likened to the Short Time Fourier Transform (STFT) with a mooving time window (Ján, 2002). The continuous wavelet transform of time series $s(t)$ with respect to the mother wavelet $\psi_{a\tau}(t)$ is defined as

$$S_{CTW}(a,\tau) = \int_{-\infty}^{\infty} s(t) \frac{1}{\sqrt{a}} \psi\left(\frac{t-\tau}{a}\right) dt, \quad a > 0, \tau \in R, \quad (6)$$

where mother wavelet takes the form $\psi_{a,\tau}(t) = \psi\left(\frac{t-\tau}{a}\right)$, τ is the time position, a is the parameter of dilatation (scale), which is related to the Fourier frequency and numerator of the fraction ensures the conservation of energy.

As stated by Ján (2002), wavelet description of time series could be better than classical Fourier spectrum for a number of time series, as it provides a good time series approximation with only a small number of spectral components. The reason is that the time series may contain sharp edges or discontinuities that require more spectral coefficients in the classical spectra, while the wavelet representation can represent the same information with sufficient accuracy with only a small number of spectral components.

To be the invertible transform, basis (mother wavelets) functions must be mutually orthogonal, have zero mean value and limited to finite time interval. That is

$$i) \quad \int_{-\infty}^{\infty} \psi_{a,\tau}(t)\, dt = 0,$$

$$ii) \quad \int_{-\infty}^{\infty} \psi_{a,\tau}^2(t)\, dt = 1, \quad (7)$$

$$iii) \quad 0 < C_\psi = \int_0^{\infty} \frac{|\Psi(\omega)|^2}{\omega} d\omega < \infty; \quad \Psi(\omega) = \int_{-\infty}^{\infty} \psi_{a,\tau}(t) e^{-i\omega t} dt,$$

where Ψ (ω) is the Fourier transform of ψ(t). There is an inverse wavelet transformation define as

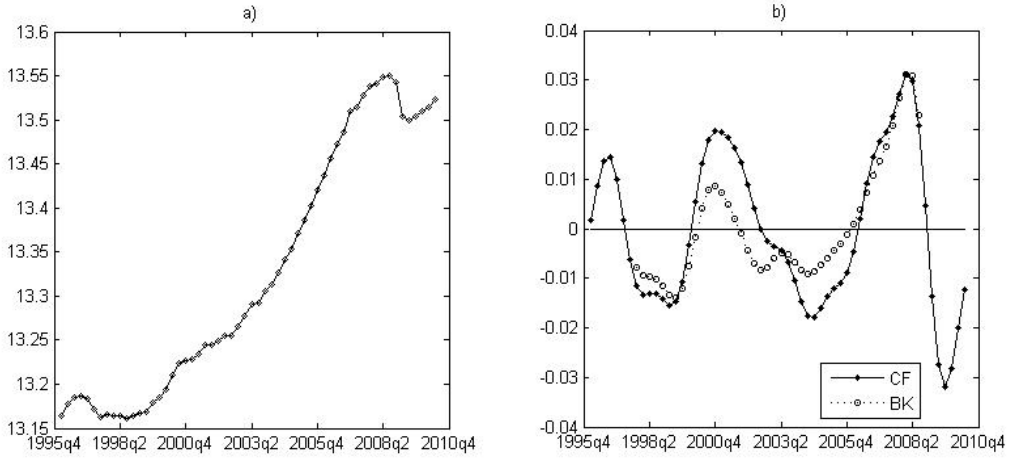
$$s(t) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi_{a,\tau}(t) S_{CWT}(a,\tau) \frac{da\,d\tau}{a^2}. \quad (8)$$

To satisfy assumptions for the time-frequency analysis, waves must be compact in time as well as in the frequency representation. There are a number of wavelets used, such as Daubeschie, Morlet, Haar or Gaussian wavelet (Grençay, Selçuk and Whitcher; 2002).

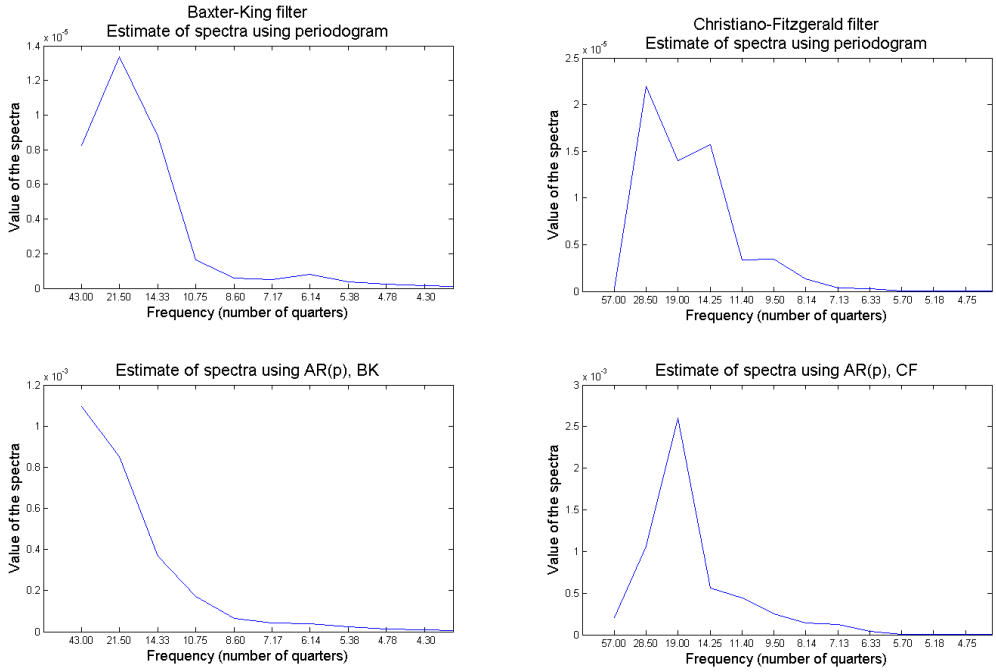
RESULTS AND DISCUSSION

For identification of business cycle data of gross domestic product (GDP) in quarterly values, millions of national currency, chain-linked volumes, reference year 2000 (including 'euro fixed' series for euro area countries), seasonally adjusted are used. The source of the data for the Czech Republic 1996/Q1–2010/Q2 (CZ) (Fig. 1a) is the free Eurostat (Eurostat) webpage and they are transformed into the natural logarithms.

Empirical analysis will be done in following steps. In the first step growth business cycles identification using Baxter-King (BK) filter and Christiano-Fitzgerald (CF) filter with pre-filtering using Hodrick-Prescott filter will be done (Fig. 1b). Setting the frequency of both band pass filters is 6 quarters for highest and 32 quarters for the lowest frequency. Consequently, dating using Bry-Boschan algorithm and calculating the length of identified cycles (peak to peak) will be added. In the second step frequency analysis will be proceed, i.e. estimation of the spetra using periodogram and autoregressive process with optimum lag order will be done. Obtained types of cycles will be tested by R. A. Fisher test (Poměnková and Maršálek, 2010), by its modification according



1: *GDP of the Czech republic (a) and corresponding growth business cycles (b)*



2: *Estimate of the spectra for growth business cycle of the CZ*

to Bolviken (2011) and and using Sarlan (2001) approach. In the third step time-frequency analysis using continuouse wavelet transformation will be concluded. For this aim Daubeschie wavelet will be used.

According to Artis, Marcellino and Proietti (2004) definition of the cycle and in the sense of the study Poměnková (2010b) calculation peak to peak cycle length in time domain was done. Results showed that in the Czech Republic only two cycles with the length of 16 and 29 quarters exists.

In the next step, analysis in the frequency domain has been proced. Firstly, verification of growth business cycles stationarity was tested. For this aim lag order for Adjusted Dickey-Fuller (ADF) test was chosen using information criteria – Akaike (AIC), the Cauchy-Schwartz (SC) and Hannah-Quine (HQC) (Green, 2008). Then Jarque-Bera test of normality (Green, 2008) and Dickey-Fuller (DF) test for white noise of obtained residue of ADF test were applied. For the Czech Republic the growth business cycles are stationary with lag order equal to 1. As was mentioned above, in order to increase the robustness of the results, two methods for the spectra estimate have been applied – periodogram and autoregressive process with optimum lag p. For optimization of this parameter information criteria mentioned above were also applied. Thus, optimum value of parametr for AR(p) process is for growth business cycle obtained using Christiano–Fitzgerald filter $p = 11$ and for growth business cycle obtained using Baxter-King filter $p = 13$. Estimated spectra were calculated using equation (4) and (7) and are showen on Fig. 2.

I: *Proportion of variation in frequency band: 6 to 32 quarters*

CF	CR		BK	
Spectrum estimate	Per.	AR	Per.	AR
Proportion	99.21%	95.92%	72.66%	56.73%

Source: Own calculation
Note: Per. – spectrum estimate using periodogram, AR – spectrum estimate using autoregressive AR(p) process

For the estimated values of the spectrum the proportions of each variance of the spectrum components relatively to the total variance of the spectrum (cumulative sum of the spectra) were also measured. As the table I showed, in the case of the CF filter within a specified range of business cycles (6 to 32 quarters) is more than 95% of the total variance, in the case of BK filter these values are significantly lower. It can therefore be concluded that the application of the CF filter to find the growth cycle for short sample sizes on the basis of calculation of the proportions seems to be appropriate, better approximating the ideal filter than in the Baxter-King filter case.

In the next stage of the analysis of estimated values of spectra were tested for its statistical significance. In the first step R. A. Fisher was applied. This test is used for testing the statistical significance of the proportion of variance in the frequency on spectrum of the total variance, while the values of the spectra are ordered in descending order. If a given spectrum component is classified as significantly involved in the total spectrum, then the total variance of the spectrum is reduced about this value for further testing. Applying this test to the CZ showed that the method is weak, because denoted as highly statistically significant (at 1% risk) also the periods, with the share of the total variance small compared to others. This situation occurred for the values of spectrum estimation using periodogram as well as using AR process. In the case of BK filter R. A. Fisher test identified as the most important periods the first four greatest values of periods.

Closer examination of graphical presentation of estimated spectrum and the variance ratio assessment, including analysis of pass-band zone of used filters outside the specified frequency range we conclude that in the case of used data sets R. A. Fisher test is rather inadvisable. The solution is to either use a stronger criterion, or to identify the growth cycles using more than two methods and results of tests to evaluate the significance of the spectrum then in their cross section. As another approach was chosen modification of R. A. Fisher test proposed by Bolviken (2011). Principle of modified test used for assessment the significance of the estimated spectral components, which are first sorted in ascending order is the calculation of the ratio between evaluated component variance to the total variance, while the predetermined number of spectrum components with the largest variance are not used for the calculation of the total variance (parameter trimming). Compared with a conventional R. A. Fisher test, every individual variance is compared to the same total variance of estimated spectrum. For details on the methodology of calculating critical parameter values and determining trimming parameter see Bolviken (2011). Another way to evaluate the spectrum estimates has been proposed by approach published in Sarlan (2001), indicating as significant the frequency corresponding to the value of spectrum, which is the peak of the spectrum followed and proceeded by spectral component statistically equivalent to zero.

The question is when to admit the existence of a cycle with specified length. If we want the results were sufficiently robust, then the consensus of identified types of cycles of lengths for different methods of spectrum estimation using the method of obtaining a sufficient quality of growth cycle did not vary substantially, even in identifying the type of cycle, or its significance. Then, there would be a cycle length allowed if the estimate proves the value of the spectrum corresponding to frequency as statistically highly significant, occurring in cross section of detrending methods. In other words, the existence of a certain period (the length of the cycle) is permitted, if the value of spectrum estimation by both methods tested by modified R. A. Fischer test is indicated as highly statistically significant (***, 1% risk), or in one case highly statistically significant and in the second statistically significant (**, 5% risk).

II: *Estimation of the spectrum of growth business cycles of CZ (frequency band: 6 to 32 quarters)*

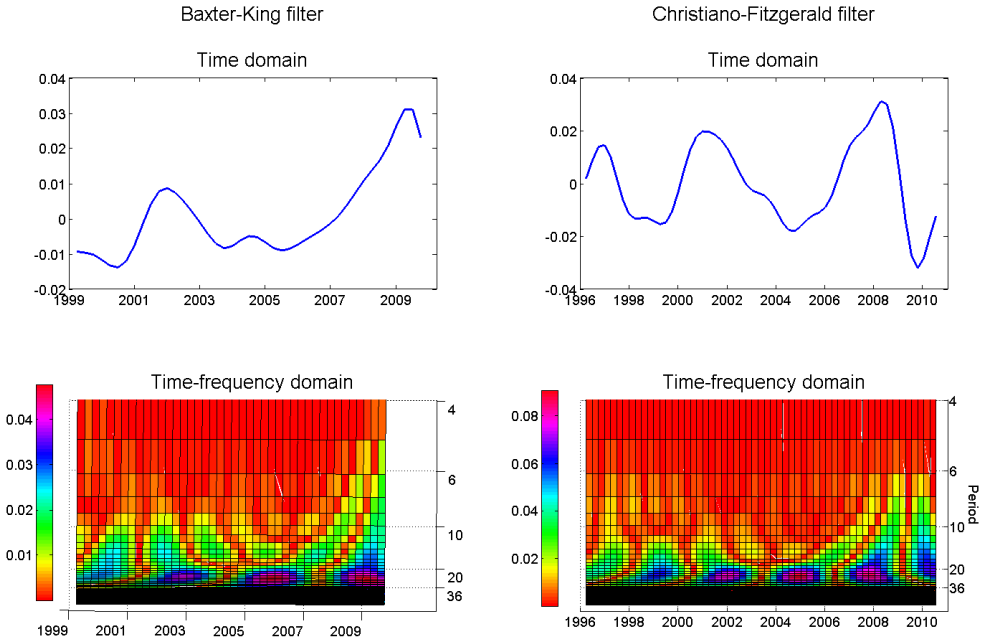
	Periods	57,00	28,50	19,00	14,25	11,40	9,50	8,14
CF	Per.		***s	***	***	***	**	
	AR		***	***s	***	***	***	**
	Periods	43,00	21,50	14,33	10,75	8,60	7,17	
BK	Per.	***	***s	***	*			
	AR	***s	***	***	***			

Source: Own calculation
Note: Statistically significant at a 1 %(***), 5%(**), 10 %(*), Periods are given in quaters, Per. – spectrum estimate using periodogram, AR – spectrum estimate using autoregressive AR(p) process, s – significant according to Sarlan

III: *Identified existing length of cycles (in years)*

Time Domain	4.0	7.3			
Frequency Domain (Bolviken)	7.1	4.8	3.6	2.9	2.4

Source: Own calculation



3: *Time-frequency representation for the Czech Republic*

In the case of the Czech Republic based on the methodology used it is possible to admit the existence of cycles of length 28.5, 19, 14.25, 11.4 and 9.5 quarters. Estimation of the growth cycle, with BK filter and subsequent calculation of the estimate of the spectrum confirmed the existence of these or cycles of closed length.

When applying the approach according to Sarlan (2010) we can admit only the cycle length of 28.5 quarters (CF, periodogram), 19 quarters (CF, AR), 21.5 quarters (BK periodogram) and 43 quarters (BK, AR). The last mentioned cycles can be understood as the result of pass-band zone of Baxter-King filter outside the band, and therefore will not be considered. Due to the frequency resolution, which in case of such a short time series is difficult, it can be concluded that the economic cycle of the Czech Republic has two nested cycles of the lengths of 28.5 and 20 quarters. However, if we even considered this type of assessment for the existing period, which is also reflected in the spectrum estimation by both methods, then the evaluation is very difficult. We could accept the length of period of 20 quarters, but confirmation of this conclusion would require additional analysis.

Reported results can also be used to assess the suitability of the applied methods. As already explained above, is inappropriate for testing estimation of the spectra R. A. Fisher test, it is better to replace it by its modification (Bolviken, 2011). Furthermore, it seems to be appropriate to use for identification of the growth business cycle Christiano-Fitzgerald filter and subsequently to apply for the spectrum estimation methods described above, which provide very similar results. In terms of performance techniques, periodogram could be recommended due to the simplicity of usage.

When comparing the results of time and frequency domain, we see in that analysis in the frequency domain revealed nested business cycles in addition to already identified cycles in the time domain. It is therefore appropriate to proceed to the next step in the analysis of time-frequency domain, thus linking the results of the two domains.

With respect to time-frequency domain approach, the wavelet analysis using the Morlet wavelet has been used. A choice of the mother wavelet was inspired by the article of Rua (2010). Resulting time-frequency representation of wavelet spectra for the Czech Republic can be seen on the figure 3 below.

Time-frequency representation of the business cycle of the Czech Republic (Fig. 3) obtained using the CF filter for detrending shows how the cycles of different lengths change – appear and disappear over the time. For the whole observed period, the longer cycles are more significant than the shorter. Only around the years 2001 and 2004, the presence of shorter cycles (10 periods) could be considered. During 2004–2007 only the very long cycles (close

to period of 36 quarters) could be identified. In time of global economic crisis, i.e. after 2008, the longer cycles are becoming more important (higher values of the wavelet spectrogram), moreover the significance of shorter cycles up to a period of about 6 quarters is also increasing. Time-frequency representation of the economic cycle of the Czech Republic (10c) identified using BK filter describes the evolution of wavelet spectra analogously. There is also a tendency of cycles to appear and disappear in time. Analysis results are very close to that obtained using the CF filter and can therefore be seen as confirmation of the previous analysis and conclusions. Due to the shortening of the input data file for the period 1997/Q3–2007/Q4 which is caused by the BK filter, the situation after 2008 is not fully reflected. Despite this, there is also an evident impact of the economic crisis.

CONCLUSION

We can conclude that the cycles are not constant over time. Cycles arise and disappear, both in terms of time, so in terms of frequency. This conclusion is consistent with Burns and Mitchel (1946), who said that *a cycle consists of expansions occuring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own.*

The type of methodical approach definitely depends on the aim of the researcher. If the time domain is applied, the researcher should work with the fact of impact of sample size and the technique of business cycle identification on the consequent work, like for example measurement of synchrony by method in the time domain. Frequency domain is appropriate to complete the business cycle description, its structure including different waves.

These two different approaches are integrated into time-frequency domain analysis. Time-frequency domain has advantage to describe behaving even small sample size in time and in frequency domain all at once. This methodological approach is very useful to understand economic activity movements in transition economies, which undergone economic liberalization and institutional changes. Small sample size and structural breaks distort frequently applied time domain analysis. The frequency domain analysis is limited by unregularity of movements in economic activity as well.

SUMMARY

The transition economies are affected by institutional changes which caused structural breaks in economic activity movements. Transformation depression or changes in foreign trade partners are only a few cases. This papers starts with business cycle analysis in time domain, using two band-pass filter, namely Baxter-King and Christiano-Fitzgerald filter. The choice of two filters was motivated by the robustness of the results. After that, dating of obtained growth cycles via Bry-Boschan algorithm was done. The results showed existence of two cycles of the length 16 and 29 quarters. Thereby, the authors described business cycle phases, i.e. expansion and recession. Unfortunately, the results may oscillate in dependency on the type of technique for growth business cycle identification as well as on the method for business cycle dating. From chosen filtering method we can say, that Baxter-King is suitable only in case were larger sample size is available, which is not the case of transition economies such the Czech Republic.

According to the theoretical framework of Baxter-King filter the loss of observation is obvious which in small sample size may cause smaller accuracy of ideal filter approximation and by this way also the worse quality of obtained business cycle. Combination of band-pass filter with dating by Bry-Boschan algorithm provides single valued results. As a disadvantage can be also denoted disability to capture cyclical movements in economic activity in the sense of nested cycles as well as cyclical behaving before/after year 2008. We are able to recognize the phase of business cycle in this year indeed, but modelling of business cycle in two separate parts is with respect two sample sizes, especially after year 2008, is very difficult.

Consequently, analysis continued in frequency domain perspective, where cyclical movements in economic activity were described by spectrum estimation. Also in this step, two methods for spectra estimate motivated by the robustness of the results was investigated, the periodogram and spectrum estimate via autoregressive process. Obtained values of spectrum estimate were tested for their significance. For the estimated values of the spectrum the proportions of each variance of the spectrum components relatively to the total variance of the spectrum were also measured. The results confirmed conclusion from time domain approach that Baxter-King filter is in the case of small sample size rather inappropriate. In case of estimated spectrum closer examination of graphical presentation of estimated spectrum and the variance ratio assessment, including analysis of pass-band zone of used filters outside the specified frequency range showed conclude that in the case of used data sets

R. A. Fisher test is rather inadvisable. The solution is to either use a stronger criterion, or to identify the growth cycles using more than two methods and results of tests to evaluate the significance of the spectrum then in their cross section. As another approach was chosen modification of R. A. Fisher test proposed by Bolviken. Then, based on the Boldviken modification it is possible to admit the existence of cycles of length 28.5, 19, 14.25, 11.4 and 9.5 quarters. Estimation of the growth cycle, with BK filter and subsequent calculation of the estimate of the spectrum confirmed the existence of these or cycles of closed length. As a disadvantage of frequency domain approach can be denoted inability to establish phases of business cycle like in time domain, also dating is not possible here. But advantage is possibility to study cyclical structure and nested cycles inside business cycle. Also frequency domain is not able to capture cyclical behaving of business cycle before/after year 2008.

When comparing the results of time and frequency domain, we see in that analysis in the frequency domain revealed nested business cycles in addition to already identified cycles in the time domain. It is therefore appropriate to proceed to the next step in the analysis of time-frequency domain, thus linking the results of the two domains.

As the last one approach frequency-domain approach was taken. Via this way, each cycle can be studied from its behavior in time as well as in frequency point of view even in small sample sizes. In the Czech Republic case we can state that the cycles are not constant over time. Cycles arise and disappear, both in terms of time, so in terms of frequency. In time of global economic crisis, i.e. after 2008, the longer cycles are becoming more important (higher values of the wavelet spectrogram), moreover the significance of shorter cycles up to a period of about 6 quarters is also increasing. Thus, even in case of small sample size we are able to see behaving before/after year 2008 which seems to be big advantage compare to both previous methodical approaches.

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