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EVALUATION OF FRACTAL DIMENSIONS FROM BITMAP IMAGE (PART: ALGORITHMS)

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Abstract

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The work assembled the complex application for evaluation of main fractal characteristics from input image. New simple modifications of evaluation of positive cells according to appropriate scale were implemented. Detail description of theory for each method and description of evaluation process was included. Filtration modification (which is able to estimate the structure which belongs only to specified scale) was introduced together with volumetric modification that allows selecting of optional ratio for determination of positive cells. Also, Richardson method and Slit-Island method were converted to algorithm form. Program was compiled for processing of the following projects about influence between fractal dimensions and mechanical properties of tested materials.

fractal dimension, algorithm, wood, program application

Evaluation of fractal characteristics is usually very complicated and time-consuming activity. For our purposes we need to form automatic process of image analysis, which will be able to automatically treat the image and provide sufficient amount of information about fracture character of image. The main aim of the work is description of the main procedures and graphical user interface of application, which was compiled for that object.

MATERIAL AND METHODS

In the work we focused on evaluation of the base fractal characteristics of fractal geometry. We used three different methods of assembling of fractal dimensions and we added other modifications to these common methods.

<u>Box-Counting method</u>

Method is based on dividing of area into regular boxes with the same length of box edge equals to δ (Ref.[2]). The length of edge depends on scale ε, whereas

$$\varepsilon = 2^{-n} \text{ for } n \in Z \qquad [1]$$

The length of edge is equal to product:

$$\delta = \varepsilon L_0 \qquad [2]$$

Whereas L_0 is equal to the total length of analysed image. In real conditions the n is limited by density of records (count of pixels) in the image. For simple evaluation of minimal scale we can assume the following relation:

$$n = \frac{\ln L_0}{\ln 2}, \qquad [3]$$

where L_0 is length of image in pixels and provides maximal value of edge size possible for image scaling (δ = pixel).

For each scale ε the appropriate pair of ln $N(\varepsilon)$ and ln (δ) can be obtained. Fractal dimension is estimated in form of slope of fitted functions for those pairs.

$$\ln N_i(\varepsilon) \approx D \ln \delta_i \quad [4]$$

D is obtained by least-square method, which is used for fitting of linear function of mentioned pairs.

$N(\varepsilon)$ is count of positive boxes in grid divided image. Positivity of box means that checked box satisfies conditions of existence of checked fractal structure (or its part) within the box. Determination of box positivity is general and it is not connected strictly to B-C method. Common way of evaluation of positivity governs by any occurrence of the structure within the image. Usually no testing of shape is realized and only existence of colour, which is different than background is sufficient reason for classification of the checked box as positive. In the following paragraphs we describe our modification in evaluation of positive boxes.

<u>Filtration method</u>

The method is based on requirement of sufficient visibility of tested structure in the checked box. The condition of visibility is satisfied when object crosses the box from one side to opposite side or from one corner to opposite corner of the box on diagonal.

<u>Volumetric method</u>

The condition of visibility is satisfied in this method when sufficient part of box includes any structure. Size of part is optional and we proposed the following relation:

$$k = \frac{V_i^p}{V_i^B} = \frac{2}{3}, \quad [5]$$

where, V_i^p is positive area (volume) and V_i^B is area (volume) of the whole i-th box.

<u>Robust method</u>

The method is on the other side of "sensitivity" of determination of the structure within the image. There is box considered as positive when the full checked box includes the structure. The method can be useful in problems with lined structure, which has decreasing thickness of lines, and when structures from specified scale to all higher scales are accepted.

<u>Slit island method</u>

In our work we used the fractal dimension in form of fractal dimensional increment according to the Mandellbrot (Ref. [1]) as:

$$D'_s = \left(\frac{\log \sum_i^n At}{\log \sum_i^n Pt} \right) - 1 \quad [6]$$

For better comparison with other methods we used the method in form (it is comparison in sense of range it is regardless to physical background):

$$D_s = D'_s + 1 \quad [7]$$

Where A_i is area of the i-th island and P_i is perimeter of the i-th island from all n such islands.

The problem of measuring of perimeter with a scaling length leads to problem of signal noise within the image. The problem is similar with noise of lower scales, which we mentioned in paragraph Introduction. From that reason the filtration modification was implemented in perimeter evaluation.

<u>Richardson method</u>

The method (Ref.[2, 3]) is representative of profilometric method which is focused on measuring of the profile length of the tested curve or profile. Estimation of fractal dimension is made by the relation:

$$D'_R = 1 \frac{\log \frac{L_t}{L_0}}{\log \eta} \quad [8]$$

For better comparison with other methods we used the method in form (it is comparison in sense of range it is regardless to physical background):

$$D_R = |D'_R - 1| \quad [9]$$

Where L_t is true length of profile measured with a scaling length η and L_0 is its projected length on the plane of anatomical slice.

The problem of measuring of the true length with a scaling length leads to problem of signal noise within the image again. From that reason the filtration modification was implemented in length evaluation.

RESULTS

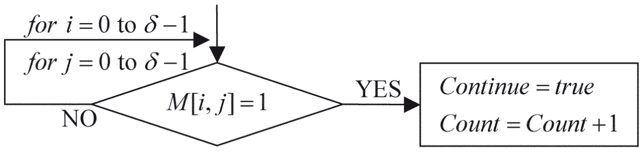
Before the image processing the input file of picture has to be converted into 1-bit format (black and white)[1]. Conversion provides the one layer matrix of negative (white = 0) and positive (black = 1) points.

[1] For evaluation of fractal dimension of fracture is recommended to smooth edges of structure within the image file

For each scale we can obtain an array with the area [$\delta \cdot \delta$]. The area can be signed as **M**. Crucial point is evaluation of count of positive matrices within the whole matrix.

<u>*Common Box-Counting method*</u>

Conventional procedure analyse each cell (pixel) of input matrix (with size according to specified scale) and when any positive pixel is found then cycle of evaluation is interrupted and analyzed matrix **M** is considered as positive and the global counter *Count* is increased. The method is very quick and suitable for very large images.

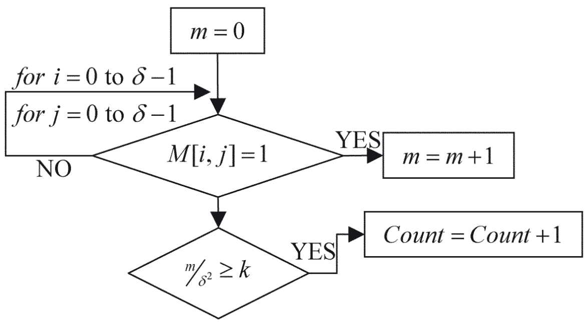


1: *Diagram of evaluation of count of positive cells in common Box-Counting method*

The global counter *Count* is equal to $N(\varepsilon)$ in mentioned way of evaluation of estimation of fractal dimension by Box-Counting method (Eq. [4]) in paragraph Methods and materials.

<u>*Volumetric method*</u>

Volumetric method rises from common Box-Counting method. On the contrary, there is summed the total count of positive cells *m* within the matrix **M** if the ratio of positive cells and rank of regular matrix **M** is greater equal than specified value *k*, the whole matrix **M** is considered as positive and the global counter *Count* is increased.



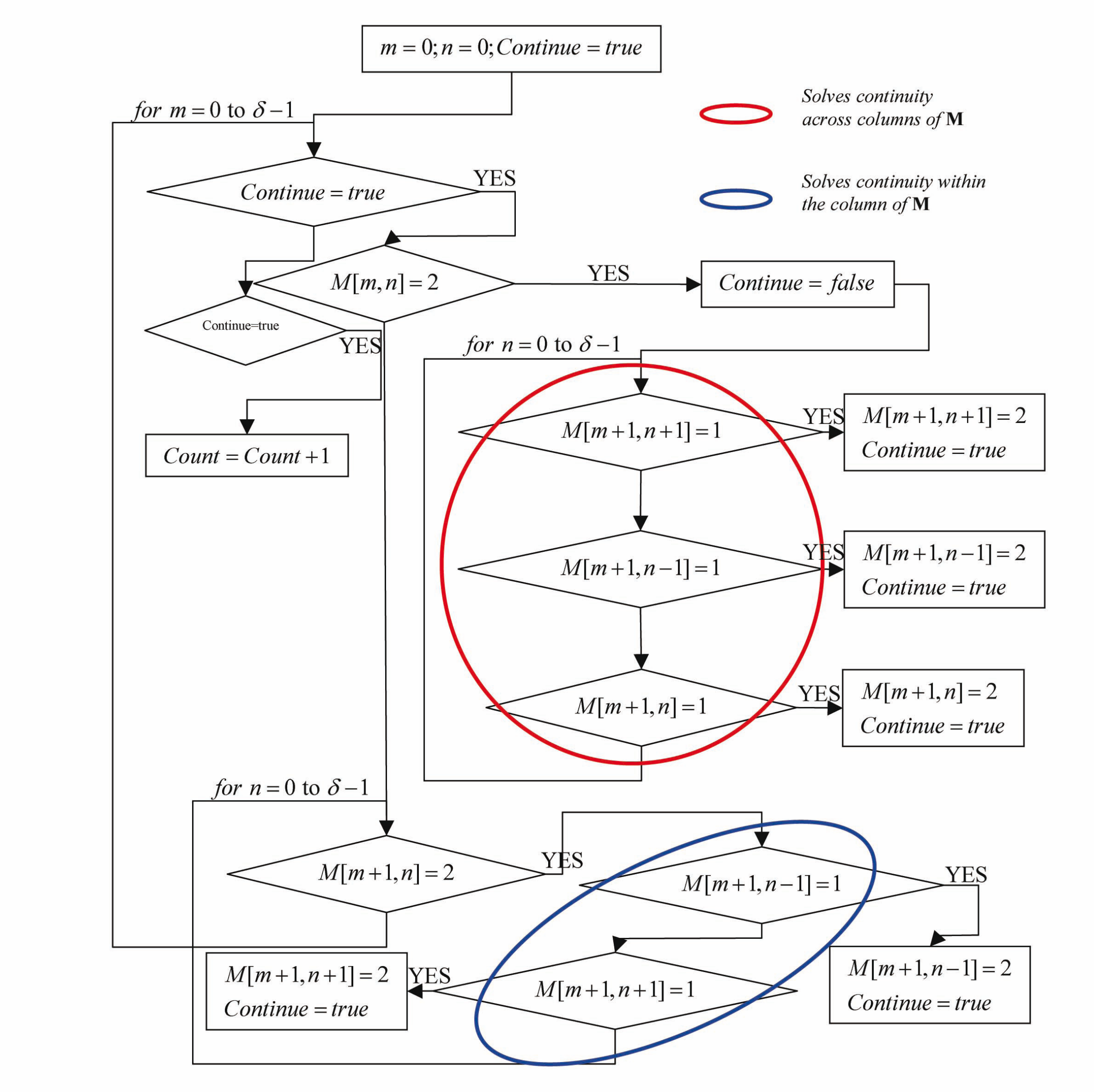
2: *Diagram of evaluation of count of positive cells in volumetric method*

The global counter *Count* is equal to $N(\varepsilon)$ in mentioned way of evaluation of estimation of fractal dimension by Box-Counting method (Eq. [4]) in paragraph Methods and materials.

<u>*Filtration method*</u>

In filtration method is the first column filled by third status (except the white and black status) (e.g. red = 2). The new state was introduced only for identification of checked parts of matrix **M**. The Fig. [3] shows only diagram of detection of crossing structure from left side of matrix M to right side of matrix. Evaluation of crossing from top to the bottom is made by simple exchange of matrix indexes ($m = n \wedge n = m$). Diagram has two parts. First, solves problem of continuity across the columns (red circle). Second, solves problem when positive sequence on next column was found, but lower cells (in the same row on the top or on the bottom according to the checked position) were not identified as positive points. Procedure checks the nearest neighbours in specified point $M[i,j]$. If positive neighbour is found then red point is placed there. The whole matrix is positive when the continue path of red points from one side to opposite side is found.

The global counter *Count* is equal to $N(\varepsilon)$ in mentioned way of evaluation of estimation of fractal di-



3: *Diagram of evaluation of count of positive cells in filtration method1*

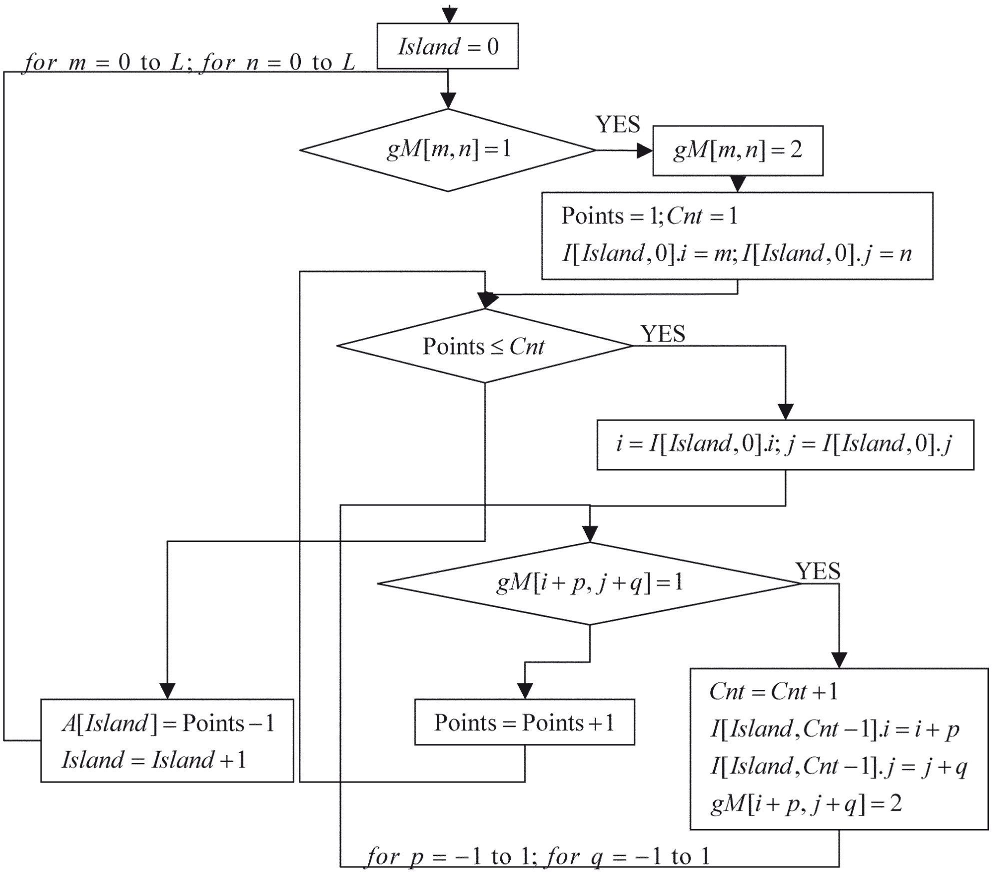
mension by Box-Counting method (Eq. [4]) and equal to L_t by Richardson method (Eq. [9]) in paragraph Methods and materials.

Richardson method

Evaluation of Richardson method is combination of mentioned procedures. Filtration process provides count of positive cells within the whole image and fractal dimension is computed due to Eq. [9].

Slit-island method

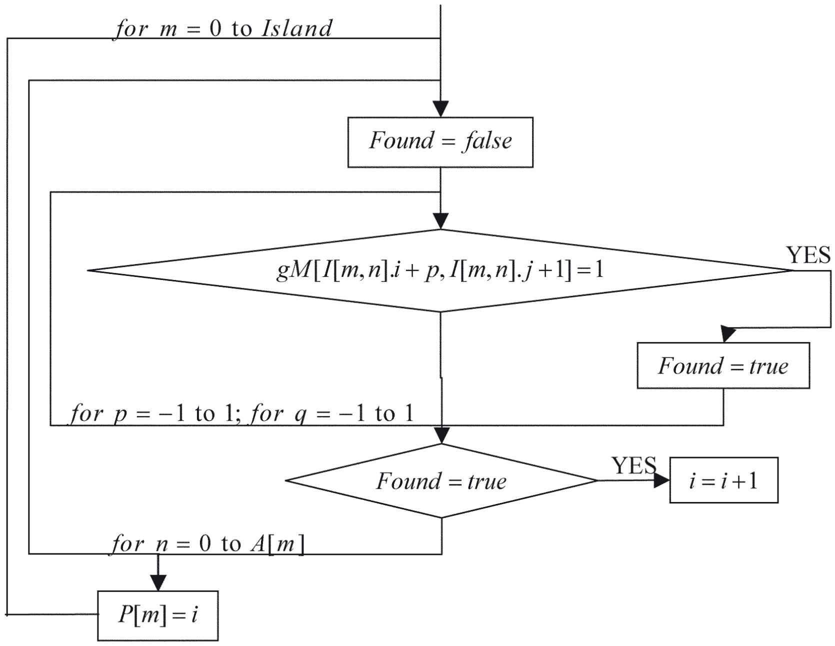
First, positive cell within the image is found and array for the first island is established (*I*[*Island*, 0]). Next, neighboring positive cells are added to that island. Positions of all positive cells within the island are saved in record *I*[*Island*, *Cnt*] . *i* and *I*[*Island*, *Cnt*] . *j*. All checked cells are marked by another status for excluding from the following seeking. Checked points for actual island are saved in variable (*Points*) and total count of points within the island is saved to vector *A*[*Island*]. Finally, next separated island is found (counter *Island* is increased) and process repeats the same way (Fig. [4]).



4: *Evaluation of stress of all separated islands*

Where
Cnt is counter of actual processed points of island with no. *Island*
I[*Island*, *Cnt*] is matrix of positions of point *Cnt* which belongs to the *Island*.
gM is global matrix of the whole image
Points are total count of actually processed *Island*.
L is length of the whole image.
Island is number of processed island.

Perimeters of island *m* are derived by checking of all points within the specified island. If they have some negative neighbours they are considered as part of island perimeter. When negative neighbours are found then counter *i* of island perimeter *m* is increased. Finally, each island *m* has determined appropriate perimeter *P*[*m*] = *i* (Fig.[5]).



5: *Evaluation of perimeters of all islands in the whole image*

From mentioned procedures (Fig. [4] and Fig. [5]) we will obtain areas of islands (**A**) and perimeters of those islands (**P**) which are sufficient for computing of fractal dimension according to the Eq. [7].

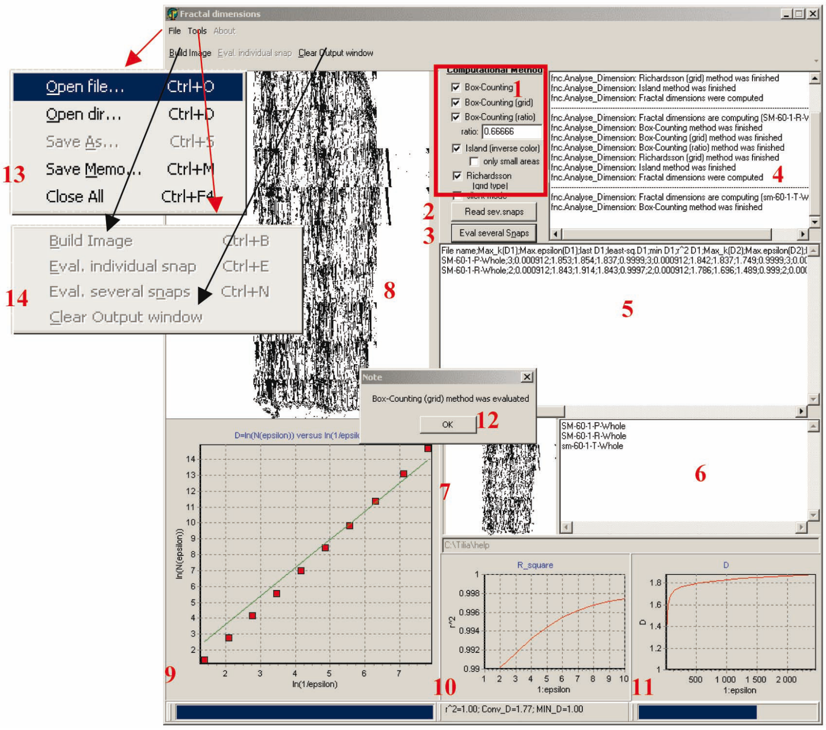
GRAPHICAL USER INTERFACE

User-friendly application for evaluation of all mentioned fractal characteristics was created. User can only define the input image(s) and application will process the appropriate image analysis without any intervention. On the contrary, user need to be educated in fractal (image) analysis and provide sufficient (appropriate) input file. Thus, some preliminary modifications have to be done before evaluation of fractal dimensions (e.g. conversion into 1-bitmap code, edge smoothing, inversion of colors for highlighting of investigated structures etc.).

Application environment (Fig.[6]) offers several active items and mostly passive information components.

SUMMARY

Assembled application is able to compute the fractal dimensions by several methods with significant decreasing of user effort. Concurrently, evaluation of reliability of fitted function by least-square method for estimation of fractal dimension was implemented. Included modifications allow new improvement of classification of self-similar structure.



6: *Application environment*

1 – User can select methods, which will be used for computation of fractal dimensions

Box-Counting is common Box-Counting method without any modification

Box-Counting (grid) is method with filtration modification

Box-Counting (ratio) is method with volumetric modification

Island is Slit-island method with filtration modification

Only small areas – when it is checked only areas smaller than average area of islands will be included

Richardson (grid) is method with filtration modification

Silent mode – allows silent mode in batch processing of large amount of images. When it is checked no window will appear and no user interaction will be requested

2, 13 – When the button is pressed the window dialog will be opened and user can select directory where image for batch processing are saved.

3, 14 – When the button is pressed the main code will be run.

4 – Output window informs about phase of computation. (Informs about finished procedures)

5 – Result window includes computed characteristics

6 – List of image files for batch processing

7, 8 – Image preview

9 – Output graph of ln $N(\varepsilon)$ and ln $(1/\varepsilon)$. It is used for fractal dimension is evaluation by least-square method. It actualizes for each running method.

10 – Output graph of coefficient of determination, which determines the relationship between the wellness of model fit and appropriate scale $1/\varepsilon$. It actualizes for each running method.

11 – Output graph of relation between estimation of fractal dimension on specified scale and scale $1/\varepsilon$

12 – Message window, which stops running of the application after each computed method

SOUHRN

Vyhodnocení fraktálových dimenzí z bitmapové předlohy (Část: Algoritmy)

Vytvořená aplikace je schopna vyhodnotit fraktálové dimenze pomocí několika základních metod fraktálové geometrie za výrazného snížení uživatelova vstupu do obrazové analýzy. Současně bylo zahrnuto vyhodnocení spolehlivosti proložené funkce metodou nejmenších čtverců pro odhad fraktálové dimenze. Implementované modifikace výpočtu fraktálové dimenze umožňují nové zlepší klasifikace sobě podobných struktur.

fraktálová dimenze, algoritmy, dřevo, programová aplikace

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